1. Let \( f(x, y) = xy(x^2 - y^2)/(x^2 + y^2) \), if \((x, y) \neq (0, 0)\) and \(f(0, 0) = 0\).
   (i) Compute \( \frac{\partial f}{\partial x}(0, 0) \) and \( \frac{\partial f}{\partial y}(0, 0) \).
   (ii) Verify that \( \frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y) \) for \((x, y) \neq (0, 0)\)
   (iii) Show that \( \frac{\partial^2 f}{\partial x \partial y}(0, 0) \neq \frac{\partial^2 f}{\partial y \partial x}(0, 0) \)

2. Let \( f(x, y, z) = x^2 + 3y^2 + 2z^2 - 2xy + 2xz \). Show that 0 is the minimum value of \( f \).

3. Let \( g(s, t) = \frac{1}{s^2 + st + t^2} e_1 + \frac{1}{(s^2 + st + t^2)^2} e_2 \) and \( \Delta = \{(s, t) : 0 < s^2 + t^2 \leq 1\} \).
   Find
   (a) \( g(\Delta) \),
   (b) the rank of \( Dg(s, t) \),
   (c) \( Jg(s, t) \)
   (d) \( g^{-1}\{e, c^2\}\)

4. Let \( g(s, t) = (e^s \cos t) e_1 + (e^s \sin t) e_2 \) and \( \Delta = \mathbb{R}^2 \).
   Find \( Jg(s, t) \). Show that \( g \) is not univalent. Find a set \( \tilde{\Delta} \) such that the restriction of \( g \) to \( \tilde{\Delta} \) is univalent and find its inverse.

5. Under what condition on \( a, b, c, d \) the following system has a unique solution in a neighborhood of \((x, y_1, y_2) = (0, 0, 0)\).
\[
\begin{align*}
ay_1 + by_2 + \sin x &= 0 \\
cy_1 + dy_2 + x \cos x &= 0
\end{align*}
\]
(hint. apply the implicit function theorem).