King Fahd University of Petroleum and Minerals
MATH 430 (Semester 111), Exam I

October 20, 2011

Exercise 1

1. Find the four zeros of $z^4 + 4$. Use those zeros to factor $z^4 + 4$ into quadratic factors with reals coefficients.

2. Find the roots of the equation $z^2 + 2z + (1 + i) = 0$.

Exercise 2

Let u and v denote the real and imaginary components of the function defined by

$$f(z) = \begin{cases} \dfrac{\bar{z}^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$$

1. Find $u(x, y)$ and $v(x, y)$.

2. Verify that f satisfies the Cauchy-Riemann equations at the origin $(0, 0)$.

3. Show that $f'(0)$ does not exist.

Exercise 3

1. Let the function $f(z) = u(r, \theta) + i\,v(r, \theta)$ be defined throughout some neighborhood of $z_0 = r_0 e^{i\theta_0}$.

Show that the Cauchy-Riemann equations can be written in the polar form

$$r\frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta}\ , \frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r} \quad \text{at } (r_0, \theta_0)$$

and $f'(z_0) = e^{-i\theta}(\dfrac{\partial u}{\partial r} + i\dfrac{\partial v}{\partial r})$ where the right-hand side is evaluated at (r_0, θ_0).

2. Let $f(z) = e^{-\theta}\cos(\ln r) + ie^{-\theta}\sin(\ln r)$ $(r > 0 \text{ and } 0 < \theta < 2\pi)$

(a) Show that f is analytic in the indicated domain.

(b) Express $f'(z)$ as a function of z.

Exercise 4

1. Find the harmonic conjugate of $u(x, y) = y^3 - 3x^2y$.

2. Write the corresponding analytic function in terms of z.