KING FAHD UNIVERSITY OF PETROLEUM AND MINERAL

Department of Mathematical Sciences

Test No. II　　　　MATH - 521　　　　Sem 111

Student #:______________ Name:______________________ | | 100 |

Show All Your Work. No Credits for Answers Not Supported by Work.

In this exam the symbols $\Im_F$, $\Im_C$, and $\Im_D$ will denote the topology of finite complement, the topology of countable complement and the discrete topology respectively.

Q1) (16 Points) Define each of the following:

a. Separated sets

b. Retract subset

c. Completely normal space

d. Heine-Borel Theorem

Q2) (16 Points) True or false. Tick as true (√) or false (X):

a. Every compact subset of a Housdorff space is closed.

b. Every closed subset of a compact space is compact.

c. Every compact Housdorff space is regular.

d. The intersection of two compact sets is compact.

e. The space $(\mathbb{R}, \Im_D)$ is connected

f. The space $(\mathbb{R}, \Im_F)$ is compact

g. The space $(\mathbb{R}, \Im_D)$ is metrizable.

h. Every metric space is Housdorff.

Q20) (12 Points) Let $\mathbb{R}$ be the set of real numbers

a. Define the Euclidean metric *e* for $\mathbb{R}^2$

b. Describe two bounded metrics on $\mathbb{R}^2$ associated with *e*.

c. What can you say about the **cardinality** of connected subsets of $\mathbb{R}^2$ with the usual topology?

d. Describe the compact subsets of $\mathbb{R}^2$ with the usual topology?

Q4) (24 Points) either prove or disprove each of the following statements:

a. If a topological space $(X, \mathfrak{I})$ has the fixed point property, and A is a retract subset of *X*, then $(A, \mathfrak{I}_{relA})$ has the fixed point property.

b. Every compact Housdorff space is regular

c. A closed and bounded subset of a metric space is compact.

Q5) (24 Points) Consider $\mathbb{R}$ with the usual topology $(\mathbb{R}, \mathfrak{I}_u)$. The topology of compact complements $\mathfrak{I}$ for $\mathbb{R}$ is defined as: $\mathfrak{I} = \{U \varepsilon \mathfrak{I}_u : U = \phi \text{ or } \mathbb{R} \text{-} U \text{ is compact}\}$.

a. Prove that if $\{O_\lambda\}$ is a $\mathfrak{I}-$ open cover for $\mathbb{R}$ then it must have a finite subcover.

b. Prove that $(\mathbb{R}, \mathfrak{I})$ is connected.

c. Prove or disprove that $(\mathbb{R}, \mathfrak{I})$ is Housdorff space.