1) Let \( x, y \) be nonzero vectors in \( \mathbb{R}^n \) such that \( \|x\|_2 = \|y\|_2 \). Show that there is a Householder reflector \( U = I_n - 2vv^T / v^Tv, v \neq 0 \) such that \( Ux = y \).

2) For \( A \in \mathbb{R}^{n \times n} \) develop an algorithm that produces Householder reflectors \( U_1, \ldots, U_{n-1} \) such that \( U_{n-1} \ldots U_1 A = L \) is lower triangle.

3) Factor \( A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ 1 & 6 \end{bmatrix} \) into a product QR where Q is an orthogonal and R is an upper triangle.

(a) by using Householder.
(b) by using Gram–Schmidt.
(c) by using given transformation.
(d) by using qr Matlab command.
(e) discuss your result in (a), (b), (c) and (d).