Black-Scholes Option pricing model: 
\[ d_1 = \frac{\ln(\frac{S_0}{K}) + (\sigma^2 t)^{\frac{1}{2}}}{\sigma\sqrt{t}} \]  
\[ d_2 = \frac{\ln(\frac{S_0}{K}) + (\sigma^2 t)^{\frac{1}{2}}}{\sigma\sqrt{t}} \]  

Price of call option at current time, \( C = S_0 \cdot \Phi(d_1) - K \cdot e^{-n\delta} \cdot \Phi(d_2) \)

Dividend Discount Model: 
\[ P = \sum_{i=1}^{\infty} \frac{D_i}{(1+i)^t} = \sum_{t=1}^{\infty} \frac{d_i}{(1+i-o(k-1,k))^t} = \sum_{t=1}^{\infty} \frac{d_i}{(1+i-o(i))^t} \]

Short selling yield: \( M(1+j) = M(1+i) + S_0 - S_1 - D \)

Capital Asset Pricing Model: \( \hat{R}_s = \alpha_s + \beta_s \hat{R}_m = R_f + \beta_s (\hat{R}_m - R_f) \)

Modified Duration: 
\[ DM = -\frac{dP}{dP} = \frac{\sum_{t=1}^{n} t \cdot K_r(1+i)^{-t}}{P} = \frac{\sum_{t=1}^{n} t \cdot K_r(1+i)^{-t}}{\sum_{t=1}^{n} K_r(1+i)^{-t}} \]

Macaulay Duration: 
\[ D = (1+i) \cdot DM \]

Approximate change in present value of a cashflow: 
\[ \frac{dP}{dP(i+h)} \approx h \frac{dP}{dP(i)} \]

Macaulay Duration of an n-year 0-coupon bond: 
\[ D = \frac{\sum_{t=1}^{n} t \cdot P \cdot r_j}{\sum_{t=1}^{n} P \cdot r_j} \]

Macaulay Duration of a portfolio: 
\[ D = -\frac{d}{dP} \sum_{k=1}^{n} \frac{P(V)_k}{X} \]

Conditions for Redington Immunization:
(i) \( PV_A|_{t=0} = PV_L|_{t=0} \)  
(ii) \( \frac{d}{dt} PV_A|_{t=0} = \frac{d}{dt} PV_L|_{t=0} \)  
(iii) \( \frac{d^2}{dt^2} PV_A|_{t=0} > \frac{d^2}{dt^2} PV_L|_{t=0} \)

Convexity = \( \frac{d}{dP} \frac{d}{dP} \frac{d}{dP} PV_A|_{t=0} \)

Portfolio is fully immunized if \( \sum A_i v^t \geq \sum L_i v^t \) for any \( i > 0 \)

PV cashflow \( \sum_{k=1}^{n} C_k(1+s_0(t_k))^{-t_k} = C_1(1+s_0(t_1))^{-t_1} + \cdots + C_n(1+s_0(t_n))^{-t_n} \)

Price at time 0 t-year 0-coupon bond \( \frac{C}{(1+s_0(t))^t} \)

\( n-1 \) year forward, one year interest rate: \( 1 + i_0(n-1,0) = \frac{(1+s_0(n-1))^{n-1}}{(1+s_0(n-1))^{n-1}} \)

(1 + s_0(n))^n = (1 + i_0(0,1))(1 + i_0(1,2)) \cdots (1 + i_0(n-1,0))

Force of Interest as Forward Rate: \( \delta = \alpha + \frac{d}{dt} \alpha t \)

\( \delta(t) = e^{\alpha t} = \frac{\Phi}{\Phi(t)} \delta_s du \)

Swap Rate: \( j \times \left[ \frac{1}{(1+s_0(1))^{n-1}} + \cdots + \frac{1}{(1+s_0(n))^{n-1}} \right] = \frac{i_0(0,1)}{(1+s_0(1))^{n-1}} + \frac{i_0(1,2)}{(1+s_0(2))^{n-1}} + \cdots + \frac{i_0(n-1,n)}{(1+s_0(n))^{n-1}} \)

At par yield \( r_n \) in (1 + s_0(n))^{-n} + r_n \cdot \sum_{k=1}^{n} (1+s_0(k))^{-k} = 1 \)

or
\( r_n = \frac{1-(1+s_0(1))^{-n}}{\sum_{k=1}^{n} (1+s_0(k))^{-k/n}} \)

IRR: Solve for \( j \) in \( \sum_{k=0}^{n} C_k v^t = 0 \)

Profitability Index \( I = \frac{\text{Present value of cash inflows}}{\text{Present value of cash outflows}} \)

Payback Period = first \( k \) in \( \sum_{s=0}^{k} C_s \leq \sum_{r=t+1}^{t} C_r \)

Discounted Payback Period = first \( k \) in \( \sum_{s=0}^{k} C_s v^t \leq \sum_{r=t+1}^{t} C_r v^t \)

Dollar-weighted \( I = B - [A + \sum_{k=1}^{n} C_k] \)

Time-weighted \( i = \left[ \frac{F_1}{A} \times \frac{F_2}{C_1 + C_2} \times \frac{F_3}{C_1 + C_2 + C_3} \times \cdots \times \frac{F_k}{C_1 + C_2 + \cdots + C_k} \right] - 1 \)

Trapezoidal rule to approximate \( \int_{a}^{b} f(x) \, dx \approx \frac{b-a}{2} \left[ f(a) + f(b) \right] \)

Descartes’ rule of signs of Polynomial \( P(x) \) for counting types of roots:
(i) \( n_{+ve} \) roots \( \leq n_{sign changes} \) in \( (C_n,C_{n-1},\ldots,C_1,C_0) \)
(ii) \( n_{-ve} \) roots \( \leq n_{sign changes} \) in \( ((-1)^n C_n,(-1)^{n-1} C_{n-1},\ldots,(-1) C_1, C_0) \)
Present value of accumulated value of Makeham’s annuity-due:

\[ P \] = \frac{Cv^n + Fr \cdot a_{n|j}}{C + (Fr - Cj) \cdot a_{n|j}} = K + \frac{2}{j}(C - K)  

\[ P = Fr^n + Fr \cdot a_{n|j} = F \] + \frac{Fr}{r - j} \cdot a_{n|j} = K + \frac{2}{j}(C - K)  

(i) \( P = F \) Bought at Par  (ii) \( P > F \) Bought at a Premium  (iii) \( P < F \) Bought at a Discount

\( t = \frac{# \text{ of days in the coupon period}}{# \text{ of days since last coupon paid}} \)  

\( P_t = P_0(1 + j)^t \)  

Price \( P_t = P_0 - t \cdot Fr. \)

Accumulated value of annuity with payments following geometric series:

\[ BV_{t+1} = BV_t(1 + j) - Fr \]

\( I_{t+1} = BV_t \times j \)

\( PR_{t+1} = Fr - I_{t+1} \)

\( BV_t = F[1 + (r - j) \cdot a_{n-t_i}] \)

\( I_t = F[j + (r - j)(1 - v_{n-k}^{-2})] \)

\( PR_t = F(r - j)v_{n-k}^{-1} \)

Level payments:

\( OB_t = LN(t + 1 - i)^t - K_{n-1}(1 + i)^{t-1} + K_2(1+i)^{t-2} - ... - K_{n-1}(1+i) - K_t \)

Prospective:

\( OB_t = K_{t+1} \times v + K_{t+2} \times v^2 + ... + K_n \times v^{n-t} \)

Level payments:

\( OB_t = LN(1 + i)^t - K_{n-1}(1 + i)^{t-1} - s_{t_i} = K(a_{n-1}(1 + i)^{-t} - s_{t_i}) = K(a_{n-1}(1 + i)^{-t} - s_{t_i}) \)

\( I_{t} = I_{t-1} - PR_{t-1} + PR_{t} \)

Sinking Fund periodic Outlay:

\( A = \sum_{s=1}^{n} A_s = \sum_{s=1}^{n} \left[ K_s + \frac{j}{d} (L_s - K_s) \right] = K + \frac{j}{d} (L - K) \)

Makeham’s m loans with scheduled repayments:

\( A_n = LN(1 + i)^{n-1} + L_i a_{n|j} = K + \frac{j}{d} (L - K) \)

Accumulated value of n-payment annuity-immediate of 1:

\( s_{n|j} = \frac{(1+i)^{n-1}}{i} \)

Present value of n-payment annuity-immediate of 1:

\( a_{n|j} = \frac{1}{i} \)

Present value of a perpetuity-immediate:

\( a_{\infty} = \frac{1}{i} \)

Annuity-due:

\( s_{n|j} = \frac{(1+i)^{n-1}}{i} \), \( a_{\infty|j} = \frac{1}{i} \)

Continuous annuities:

\( \bar{a}_{n|j} = \int_0^n v^t dt = \frac{1-e^{-nt}}{i} \), \( \bar{a}_{n|j} = \int_0^n v^t dt = \frac{1-e^{-nt}}{i} \)

Present value of n-term mthly payable annuity-immediate of 1/m:

\( a_{(m)n|j} = \frac{1}{i} - \frac{v^n}{i^{m(1+i)}} = a_{n|j} \times \frac{1}{i^{m(i)}} \)

Present value of annuity with non-level payments:

\( v(1+r)v^2 + ... + (1+r)^{n-2}v^n + (1+r)^{n-1}v^n = v + v + ... + v = nv \)

Accumulated value of annuity with payments following geometric series:

\( \frac{1-(1+i)^n}{i-(1+r)}(1+i)^n = \frac{1}{i-(1+r)}(1+r)^n \)

Dividend discount model for present value of a stock:

\( K \)

\( n \)-payment increasing annuity-immediate: \( (Is)_{n|j} = \frac{\bar{s}_{n|j}^{n-1}}{i} \)

\( (Ia)_{n|j} = \frac{\bar{a}_{n|j}^{n-1}}{i} \)

\( n \)-payment increasing perpetuity-immediate:

\( (Ia)_{\infty} = \frac{\bar{a}_{\infty}^{n-1}}{i} = \frac{1}{i} + \frac{1}{i} \)

\( n \)-payment decreasing annuity-immediate:

\( (Ds)_{n|j} = n \bar{s}_{n|j}^{n-1} \), \( (Da)_{n|j} = n \bar{a}_{n|j}^{n-1} \)

Capitalized Cost:

\( C = P + \frac{Pr-S_{n|j}}{\bar{s}_{n|j}} + \frac{M}{i} \)

Periodic Charge = (Capitalized Cost) \times i = \left( P + \frac{Pr-S_{n|j}}{\bar{s}_{n|j}} + \frac{M}{i} \right) \times i = Pi + \frac{Pr-S_{n|j}}{\bar{s}_{n|j}} + M \)

Depreciation Methods

1) Declining Balance (compound discount) Method:

\( P_t = P_0 \times (1 - d)^t \)  

\( D_t = P_0 \times (1 - d)^{t-1} \cdot d \)

2) Depreciation - Straight Line Method:

\( P_t = P_0 - t \times \frac{1}{n} \times (P_0 - P_n) \)  

\( D_t = \frac{1}{n}(P_0 - P_n) \)
3) Depreciation - Sum of Year Digits Method: 

\[ S_k = 1 + 2 + \cdots + k = \frac{k(k+1)}{2} \]

\[ P_t = P_n + \frac{S_{n-1}}{S_n} \times (P_0 - P_n) \quad D_t = \frac{n-t+1}{S_n} \times (P_0 - P_n) \]

4) Depreciation - Compound interest Method: 

\[ P_t = P_0 S^n t S^n (P_0 P_n) \quad D_t = \frac{(1+i)^{n-t} - 1}{S_n} \times (P_0 - P_n) \]

\[ i_{t+1} = \frac{A(t+1) - A(t)}{A(t)}, \quad A(t) = A(0)a(t) \]

a) \( a(t) = (1 + i)^t \)  \hspace{1em} Compound interest accumulation factor

b) \( a(t) = 1 + it \)  \hspace{1em} Simple interest accumulation factor

\[ v = \frac{1}{1 + i}, \quad 1 + i = \left[ 1 + \frac{i(m)}{m} \right]^m \quad i^{(m)} = m \left[ (1 + i)^{1/m} - 1 \right] \quad i^{(\infty)} = \ln(1 + i) \]

\[ d = \frac{A(1) - A(0)}{A(1)} = \frac{i}{r+\lambda} \quad i = \frac{d}{1+d} \]

a) \( (1 - d)^t \)  \hspace{1em} Compound discount factor

b) \( 1 - dt \)  \hspace{1em} Simple discount factor

\[ 1 - d = \left[ 1 - \frac{d^{(m)}}{m} \right]^m \quad d^{(\infty)} = -\ln(1 - d) \]

\[ \delta_t = \frac{A(t)}{A(t)} \quad A(n) = A(0)e^{\int_0^n \delta_s ds}, \quad 1 + x + x^2 + x^3 + \cdots + x^k = \frac{1-x^{k+1}}{1-x} = \frac{x^{k+1}-1}{x-1} \]