1. A project requires an initial capital outlay of 30,000 and will return the following amounts (paid at the ends of the next 5 years):

14,000, 12,000, 6,000, 4,000, 2,000

Solve each of the following.

(a) Internal rate of return

The IRR is the solution of the equation

\[ 30,000 = \frac{14,000}{1+i} + \frac{12,000}{(1+i)^2} + \frac{6,000}{(1+i)^3} + \frac{4,000}{(1+i)^4} + \frac{2,000}{(1+i)^5} \]

The solution is \( i = 0.1203 \)

(b) Modified internal rate of return assuming a cost capital of 10% per year.

The MIRR is \( j \), which is the solution of the equation

\[ 30,000(1 + j)^5 = 14,000(1.1)^4 + 12,000(1.1)^3 + 6,000(1.1)^2 + 4,000(1.1) + 2,000(1.1) \]

The solution is \( j = 0.1081 \)

(c) Net present value based on a cost of capital of 10% per year.

\[ NPV = -30,000 + \frac{14,000}{1.1} + \frac{12,000}{(1.1)^2} + \frac{6,000}{(1.1)^3} + \frac{4,000}{(1.1)^4} + \frac{2,000}{(1.1)^5} = 1,126 \]

(d) The payback period

\[-30,000 + 14,000 = -16,000 < 0,\]
\[-16,000 + 12,000 = -4,000 < 0,\]
\[-4,000 + 6,000 = 2,000 > 0,\]

The project breaks during the third year.
(e) The discounted payback period assuming a cost capital of 10% per year

\[-30,000 + \frac{14,000}{(1.1)^1} = -17,273 < 0\]
\[-17,273 + \frac{12,000}{(1.1)^2} = -7,356 < 0\]
\[-7,356 + \frac{6,000}{(1.1)^3} = -2,848 < 0\]
\[-2,848 + \frac{4,000}{(1.1)^4} = -116 < 0\]
\[-116 + \frac{2,000}{(1.1)^5} = -1,126 > 0\]

The project break during the fifth year

(f) The profitability index

The profitable index is

\[I = \frac{12,000 + 12,000 + 6,000 + 4,000 + 2,000}{30,000} = 1.0375\]

2. You are given the following information about an investment account:

<table>
<thead>
<tr>
<th>Date</th>
<th>Value Immediately Before Deposit</th>
<th>Deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>July 1</td>
<td>12</td>
<td>X</td>
</tr>
<tr>
<td>December 31</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.

Time weighted: \[\frac{12}{10} \times \frac{X}{12+X} - 1 = 0 \rightarrow X = 60\]
Dollar weighted: \[10(1 + i) + 60(1 + \frac{1}{2}i) = 60 \rightarrow i = -0.25\]

3. A line of credit loan of 10,000 is to be used for investment purposes. There are two investment alternatives. The first will provide payments of 3000 each year for 10 years starting one year from now. The second will provide payments of 8000 two years and five years from now, and 7000 seven years and ten years from now. The investor plans to deposit all proceeds from the investment into the line of credit account. When there is a balance owed in the account, interest is charged at 15% per year, and when there is surplus
in the account interest is credited at 9% per year. Find the account balance after 10 years for each investment alternative.

Suppose that \( t \) is the point at which the balance in the account first become positive. Then \( 16,147 = -10,000(1.15)^t \cdot (1.09)^{10-t} + X \cdot s_{t \cdot 15} \cdot (1.09)^{10-t} + X \cdot s_{10-t} \cdot 09 \)

From Example 5.6 we see from Investment 1 that \( X \leq 3000 \) so that \( t \geq 5 \). Try \( t = 5 \) : \( X = 2878.85 \)
But then balance at time 5 is \( -10,000(1.15)^5 + 2878.85 \cdot s_{5 \cdot 15} = -703.28 \), which is contrary to our assumption that \( t = 5 \). Try \( t = 6 \) : \( X = 2882.37 \)
Then balance at \( t = 6 \) is \( -10,000(1.15)^6 + 2878.85 \cdot s_{6 \cdot 15} = 2100.88 \) and then balance at \( t = 5 \) is \( -10,000(1.15)^5 + 2878.85 \cdot s_{5 \cdot 15} = -703.27 \)

Thus, \( t = 6 \) is the first point at which the balance is positive, \( X = 2882.37 \)