Quiz 5 ch7 AS201: Financial Mathematics **FORM A** Dr. Mohammad H. Omar

Name____ ID#:__________ Serial #:___

Instructions. Mobile calculators or communicable devices are disallowed. Use a scientific calculator or preferably an approved SOA financial calculator. Write important steps to arrive at the solution of the following problems.

1. Under the current market conditions Bond 1 has a price (per 100 of face amount) of 88.35 and a Macaulay duration of 12.7, and Bond 2 has a price (per 100 of face amount) of 130.49 and Macaulay duration of 14.6 . A portfolio is created with a combination of face amount F_1 of Bond 1 and face amount F_2 of Bond 2. The combined face amount of the portfolio is $F_1 + F_2 = 100$, and the Macaulay duration of the portfolio is 13.5. Find F_1, F_2, and the portfolio value.

   (Work shown 10 points)

   $P_1 = F_1 * 88.35/100$ and $P_2 = F_2 * 130.49/100$

   The Macaulay duration of the portfolio is $\dfrac{P_1 \times D_1 + P_2 \times D_2}{P_1 + P_2}$

   $= \dfrac{F_1 \times 0.8835 \times 12.7 + F_2 \times 1.3049 \times 14.6}{F_1 \times 0.8835 + F_2 \times 1.3049} = 13.5.$

   So $F_1 \times 0.8835 \times 12.7 + F_2 \times 1.3049 \times 14.6 = 13.5(F_1 \times 0.8835 + F_2 \times 1.3049)$

   $\rightarrow 11.22F_1 + 19.052F_2 = 11.927F_1 + 17.616F_2 \rightarrow F_2 = 0.49234F_1$

   Since $F_1 + F_2 = 100$, we solve the two equations to get $F_1 = 67.01$ and $F_2 = 32.99$

   The portfolio value is $67.01\,(0.8835) + 32.99\,(1.3049) = 102.25$

2. Liability payments of 100 each are due to be paid in 2, 4 and 6 years from now. Asset cashflow consists of A_1, in 1 year and A_5 in 5 years. The yield for all payments is 10%. An attempt is made to have the asset cash flow immunize the liability cashflow by matching present value and duration.

   (a) Find A_1, and A_5.

   (b) Determine whether or not the conditions for Redington immunization are satisfied.

   (Work shown 10 points)

   (a) $\frac{A_1}{1+i} + \frac{A_5}{(1+i)^5} = 100\left(\frac{1}{(1+i)^2} + \frac{1}{(1+i)^4} + \frac{1}{(1+i)^6}\right)$ where $i_0 = 0.10$.

   first condition: $\frac{A_1}{1.1} + \frac{A_5}{(1.1)^5} = 100\left(\frac{1}{(1.1)^2} + \frac{1}{(1.1)^4} + \frac{1}{(1.1)^6}\right) \rightarrow \frac{A_1}{1.1} + \frac{A_5}{(1.1)^5} = 207.39$

   $\frac{A_1}{1.1} = 207.39 - \frac{A_5}{(1.1)^5}$

   For second condition,

   $\frac{d}{di}\left(\frac{A_1}{1+i} + \frac{A_5}{(1+i)^5}\right) = \frac{d}{di}\left(100\left(\frac{1}{(1+i)^2} + \frac{1}{(1+i)^4} + \frac{1}{(1+i)^6}\right)\right)$

   $\rightarrow -\frac{A_1}{(1+i)^2} - 5\frac{A_5}{(1+i)^6} = -100\left(\frac{2}{(1+i)^3} + \frac{4}{(1+i)^5} + \frac{6}{(1+i)^7}\right)$

   Note that with an implied multiplication of 1.1

   $\rightarrow \frac{A_1}{1.1} + \frac{5A_5}{(1.1)^5} = 100\left(\frac{2}{(1.1)^2} + \frac{4}{(1.1)^4} + \frac{6}{(1.1)^6}\right) \rightarrow \frac{A_1}{1.1} + \frac{5A_5}{(1.1)^5} = 777.18$

   $\rightarrow 207.39 - \frac{A_5}{(1.1)^5} + \frac{5A_5}{(1.1)^5} = 777.18$

Solving for A_1, and A_5 results in $A_1 = 71.44$ and $A_5 = 229.41$,

(b) $\sum \frac{d}{di^2}\left(A(1+i)^{-t}\right) > \sum \frac{d}{di^2}(L(1+i)^{-t}) \rightarrow \sum \frac{(t+1)tA_t}{(i+1)^{t+2}} > \sum \frac{(t+1)tL_t}{(i+1)^{t+2}}$

$\frac{2A_1}{1.1^3} + \frac{6 \cdot 5A_5}{(1.1)^7} = \frac{2}{1.1^3}(71.44) + \frac{6 \cdot 5}{(1.1)^7}(229.41) = 3639.1, \quad 100\left(\frac{3*2}{(1.1)^4} + \frac{5 \cdot 4}{(1.1)^6} + \frac{7 \cdot 6}{(1.1)^8}\right) = 3498.1$

Since $3639.1 > 3498.1$, Redington immunization is satisfied at 10%.

or $\frac{2A_1}{1.1} + \frac{6 \cdot 5A_5}{(1.1)^5} = \frac{2}{1.1}(71.44) + \frac{6 \cdot 5}{(1.1)^5}(229.41) = 4403.3, \quad 100\left(\frac{3 \cdot 2}{(1.1)^2} + \frac{5 \cdot 4}{(1.1)^4} + \frac{7 \cdot 6}{(1.1)^6}\right) = 4232.7$

Since $4403.3 > 4232.7$, Redington immunization is satisfied at 10%.

or Simply (as seen in your book), condition 2 and $\sum \frac{d}{di^2}\left(A(1+i)^{-t}\right) > \sum \frac{d}{di^2}(L(1+i)^{-t}) \rightarrow \sum \frac{A_t t^2}{(i+1)^t} > \sum \frac{L_t t^2}{(i+1)^t}$

$\frac{A_1}{1.1} + \frac{5^2 A_5}{(1.1)^5} = \frac{1}{1.1}(71.44) + \frac{5^2}{(1.1)^5}(229.41) = 3626.1, \quad 100\left(\frac{2^2}{(1.1)^2} + \frac{4^2}{(1.1)^4} + \frac{6^2}{(1.1)^6}\right) = 3455.5$

Since $3626.1 > 3455.5$, Redington immunization is satisfied at 10%

A note about first and second derivatives versus Redington immunization conditions

(This discussion is NOT in your book or your future AS288 manuals but for those who are **intellectually curious**)

Condition 1: $\sum A(1+i)^{-t} = \sum L(1+i)^{-t}$

Condition 2: $\sum \frac{d}{di}\left(A(1+i)^{-t}\right) = \sum \frac{d}{di}(L(1+i)^{-t})$

$$\sum -A_t \frac{t}{(1+i)^{t+1}} = \sum -L_t \frac{t}{(1+i)^{t+1}}$$

simplifies to $\sum A_t \frac{t}{(1+i)^t} = \sum L_t \frac{t}{(1+i)^t}$

Condition 3: $\sum \frac{d}{di^2}\left(A(1+i)^{-t}\right) > \sum \frac{d}{di^2}(L(1+i)^{-t})$

$$\sum \frac{d}{di}\left(-A_t \frac{t}{(1+i)^{t+1}}\right) > \sum \frac{d}{di}\left(-L_t \frac{t}{(1+i)^{t+1}}\right)$$

$$\sum \frac{(t+1)tA_t}{(1+i)^{t+2}} > \sum \frac{(t+1)tL_t}{(1+i)^{t+2}}$$

$$\sum \frac{t^2 A_t}{(1+i)^{t+2}} + \sum \frac{tA_t}{(1+i)^{t+2}} > \sum \frac{t^2 L_t}{(1+i)^{t+2}} + \sum \frac{tL_t}{(1+i)^{t+2}}$$

simplifies to $\sum \frac{t^2 A_t}{(1+i)^t} + \sum \frac{tA_t}{(1+i)^t} > \sum \frac{t^2 L_t}{(1+i)^t} + \sum \frac{tL_t}{(1+i)^t}$

$\sum \frac{A_t t^2}{(1+i)^t} > \sum \frac{L_t t^2}{(1+i)^t}$ since $\sum A_t t v^t = \sum L_t t v^t$ from Redington's condition no 2.