

1. If  $C$  is a number that satisfies the conclusion of the mean value theorem when applied to  $f(x) = \ln x$  on  $[1, e]$ , then  $C =$

a)  $e - 1$

b)  $\frac{1}{e - 2}$

c)  $\frac{e + 1}{2}$

d)  $\frac{e}{2}$

e)  $\frac{4}{e + 1}$

2. Suppose that  $f$  is differentiable on  $\mathbb{R}$  and satisfies  $2 \leq f'(x) \leq 6$  for all values of  $x$ . Then  $a \leq f(5) - f(3) \leq b$  where  $b - a =$

a) 8

b) 7

c) 6

d) 5

e) 4

3. The absolute maximum of  $f(x) = \frac{\ln x}{x^2}$  on the interval  $[1, e]$  is

a)  $\frac{1}{2e}$

b)  $\frac{1}{e^2}$

c)  $\frac{1}{2}$

d)  $e$

e)  $1$

4.  $\lim_{x \rightarrow +\infty} \left(1 + \frac{3}{x} + \frac{4}{x^2}\right)^x$  is equal to

a)  $e^3$

b)  $1$

c)  $\ln 3$

d)  $e^4$

e)  $\ln 4$

5. If  $\cosh x = \sqrt{2}$  and  $x < 0$ , then  $\coth x + \operatorname{csch} x$  is equal to

- a)  $-1 - \sqrt{2}$
- b)  $-1 + \sqrt{2}$
- c)  $1 - \sqrt{2}$
- d)  $1 + \sqrt{2}$
- e)  $\frac{-1}{\sqrt{2}}$

6. If there are real numbers  $a$  and  $b$  such that  $\lim_{x \rightarrow -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2} = b$ , then  $a + b =$  is equal to

- a) 14
- b) 15
- c) 16
- d) 13
- e) 17

7. If  $4x - 9 \leq f(x) \leq x^2 - 4x - 2$  for  $x \geq 0$  and  $2x \leq g(x) \leq x^4 - x^2 + 2$  for all  $x$ . then  $\lim_{x \rightarrow 1} \frac{f(x) - g(x)}{f(x) + g(x)} =$

a)  $\frac{7}{3}$

b)  $-\frac{7}{3}$

c)  $\frac{3}{7}$

d)  $-\frac{3}{7}$

e) 0

8. If  $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x^2 - 4} = 4$  where  $f(x)$  is defined on  $\mathbb{R}$ , and  $g(x) = \frac{f(x)e^x}{1-x}$ , then  $g'(2) =$

a)  $-16e^2$

b)  $16e^2$

c)  $-4e^2$

d)  $4e^2$

e)  $-f(2)e^2$

9. If  $y = ax + b$  and  $y = cx + d$  are the equations of the tangent lines to the curve  $y = \frac{x-1}{x+1}$  that are parallel to the line  $x - 2y = 2$ . Then  $a + b + c + d =$

- a) 4
- b) 5
- c) 3
- d)  $-3$
- e)  $-4$

10. If  $f(x) = \sin(x^2)$ , then  $f'(45^0) =$

- a)  $\frac{\pi}{2} \cos \frac{\pi^2}{16}$
- b)  $\frac{\pi}{4}$
- c)  $90 \cos \frac{\pi^2}{16}$
- d) 45
- e)  $\cos \frac{\pi^2}{16}$

11. If the tangent line to the parabola  $y = x^2$  is perpendicular to the tangent line of the ellipse  $a^2y^2 + x^2 = 1$  at each point where the parabola and the ellipse intersect, then  $a^2 =$

- a) 2
- b)  $\frac{1}{4}$
- c) 4
- d) 3
- e)  $\frac{1}{2}$

12. A farmer has 4 km of fencing and wants to fence off a field that borders a straight river. The shape of the field is a rectangle ABDE with one side along the river, with an adjoining section in the shape of an equilateral triangle BCD. He needs no fence along the river. The fencing is placed only along the outer perimeter ABCDE of the field; see the diagram. What is the distance from the river to the furthest point, C, of the field with the largest area.

- a)  $\frac{4}{4 - \sqrt{3}}$
- b)  $\frac{4}{4 + \sqrt{3}}$
- c)  $\frac{2}{4 - \sqrt{3}}$
- d)  $\frac{2}{2 - \sqrt{3}}$
- e)  $\frac{3}{3 + \sqrt{6}}$

13. A linear approximation gives  $(1.02)^{10} \approx$

- a) 1.2
- b)  $1 + \frac{\ln 10}{50}$
- c)  $1.2 + \ln 10$
- d) 1.002
- e)  $0.02 + \ln 10$

14. If  $x^y = y^x$ , then find  $\frac{dy}{dx}$

- a)  $\frac{xy \ln y - y^2}{xy \ln x - x^2}$
- b)  $\frac{y}{x} \left( \frac{y}{x} - \ln y \right)$
- c)  $-\frac{y^2}{x(x - y \ln x)}$
- d)  $\frac{y}{x} \left( \frac{y}{x^2} + \ln y \right)$
- e)  $\frac{y^2}{x(x + y \ln x)}$

15.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} =$

- a)  $-3$
- b)  $3$
- c)  $9$
- d)  $6$
- e)  $+\infty$

16. Find the constants A and B such that

$$f(x) = \begin{cases} A, & x \leq 1 \\ \ln(x^4 - 1) - \ln(x - 1), & 1 < x \leq 3 \\ \ln B, & x > 3 \end{cases}$$

is continuous everywhere.

- a)  $A = 2 \ln 2, B = 40$
- b)  $A = \ln 2, B = 20$
- c)  $A = \ln 2, B = 40$
- d)  $A = \ln 2, B = 10$
- e)  $A = 20, B = \ln 3$



17. Let  $f(x) = \frac{8x^3}{(x-1)^3}$ . Which one of the following statements is correct

- a)  $f(x)$  is concave up on  $(-1, 0) \cup (1, +\infty)$  and concave down on  $(-\infty, -1) \cup (0, 1)$
- b)  $f(x)$  has one local minimum and no local maximum
- c)  $f(x)$  has an inflection point only at  $x = -1$
- d)  $f(x)$  is increasing everywhere
- e)  $f(x)$  is concave up on  $(1, +\infty)$  and concave down on  $(-\infty, 0)$

18.  $\lim_{x \rightarrow 0} \frac{(e^{x^2} - 1) \sin 2x}{x(1 - \cos x)} =$

- a) 4
- b) 2
- c) 1
- d)  $e^2$
- e) 0

19. Let  $f(x) = \frac{x^3}{9 - x^2}$ . Which statement is correct?

- a)  $f(x)$  has a local minimum at  $-\sqrt{27}$  and a local maximum at  $\sqrt{27}$
- b)  $f(x)$  has a local minimum at 0
- c)  $f(x)$  has a local maximum at 0
- d)  $f(x)$  has a local minimum at  $\sqrt{27}$  and a local maximum at  $-\sqrt{27}$
- e)  $f(x)$  has no local minimum and no local maximum

20.  $f(x) = \frac{x^3}{9 - x^2}$  has a slant asymptote given by,

- a)  $y = -x$
- b)  $y = x$
- c)  $y = 3x$
- d)  $y = -9x$
- e) Does not have a slant asymptote

21. From the plots of  $f(x)$  and  $g(x)$  shown below, what is the value of  $\lim_{x \rightarrow 1} \frac{f(x)}{g(x)}$ ?

- a) -2
- b) 0
- c) 1
- d) 2
- e)  $\infty$

22. A rectangular poster is to have an area of  $1600 \text{ cm}^2$  with 4 *cm* margins at the bottom and sides and 6 *cm* margin at the top. What are the dimensions of the poster with the largest possible printed area?

- a) width =  $16\sqrt{5} \text{ cm}$  and height =  $20\sqrt{5} \text{ cm}$
- b) width =  $8\sqrt{5} \text{ cm}$  and height =  $40\sqrt{5} \text{ cm}$
- c) width =  $32\sqrt{5} \text{ cm}$  and height =  $10\sqrt{5} \text{ cm}$
- d) width =  $64\sqrt{5} \text{ cm}$  and height =  $5\sqrt{5} \text{ cm}$
- e) width =  $4\sqrt{5} \text{ cm}$  and height =  $80\sqrt{5} \text{ cm}$

23. We use Newton's method to find  $\sqrt[8]{2}$  with an initial approximation  $x_1 = 1$ . If  $x_2$  and  $x_3$  are the next two approximations, then  $x_3 - x_2$  is equal to

a)  $\frac{2 - (\frac{9}{8})^8}{8(\frac{9}{8})^7}$

b)  $\frac{2 + (\frac{9}{8})^8}{8(\frac{9}{8})^7}$

c)  $\frac{2 - (\frac{9}{8})^8}{(\frac{9}{8})^7}$

d)  $\frac{-(\frac{9}{8})^8}{4(\frac{9}{8})^7}$

e)  $\frac{2 - (\frac{9}{8})^7}{8(\frac{9}{8})^8}$

24. If  $f'(x) = \frac{2}{1+x} - \frac{2}{\sqrt{1-x^2}}$  with  $f(0) = 0$ , then  $f(1) =$

a)  $2 \ln 2 - \pi$

b)  $\ln 2 + \pi$

c)  $\ln 2$

d)  $2 \ln 2 + \frac{\pi}{2}$

e)  $\frac{\pi}{2}$

25. The acceleration of a particle moving along a straight line is given by  $a(t) = 2t - 9 \text{ m/s}^2$ . If at time  $t = 0$  seconds its velocity is  $18 \text{ m/s}$ , and its displacement is  $0 \text{ m}$ , then the total distance moved by the particle in the time interval  $[0, 6]$  second is,

- a) 27 m
- b) 18 m
- c) 22.5 m
- d) 45 m
- e) 63 m

26. If  $f(x) = \frac{\cot(x)}{g(x)}$ , and  $f' \left( \frac{\pi}{4} \right) = -1$ , and  $g \left( \frac{\pi}{4} \right) = 1$ , then  $g' \left( \frac{\pi}{4} \right)$  is equal to

- a) -1
- b) -2
- c) 0
- d) 1
- e) 2

27. A particle is moving along the curve  $y = xe^{-x}$ . As the particle passes through the point  $(1, 1/e)$  its  $x$ -coordinate increases at a rate of  $4 \text{ m/s}$ . The rate of change of the distance from the origin to the particle at that instance is

- a)  $\frac{4e}{\sqrt{e^2 + 1}}$   
b)  $\frac{e}{\sqrt{e^2 + 1}}$   
c)  $\frac{e}{\sqrt{e^2 - 1}}$   
d)  $\frac{4e}{\sqrt{e^2 - 1}}$   
e)  $\frac{4}{\sqrt{1 + e}}$

28. The side of a cube is measured and found to be 20 cm with a maximum possible error of 0.5 cm. What is the maximum possible percentage error in the volume of the cube?

- a) 7.5%  
b) 5.5%  
c) 2.5%  
d) 5%  
e) 8%