MATH 101.10 (112)

Quiz 2 (Sects. 2.4-2.6) Duration: 20mn

Name: ID number:

1.)(4pts)Let $f(x) = \begin{cases} 2a+bx, & \text{if } x>3, \\ 4, & \text{if } x=3. \\ 2b-ax^2, & \text{if } x<3 \end{cases}$ Find the values of a and b that make f continuous everywhere.

2.)(3pts)Using the $\epsilon-\delta$ definition of limit, prove that limit $\lim_{x\to 1}(2x+1)=3$.

3.)(3pts)Find all horizontal asymptotes of $y=\frac{\sqrt{x^2+1}}{x}$.

1°) $\lim_{x\to 3^-} f(x) = f(3) = \lim_{x\to 3^+} f(x)$

If this condition is satisfied, the $f(x)$ will be continuous at $x=3$.

$$\begin{matrix} -3 \\ 2 \end{matrix}\begin{cases} 2b-9a=4 \\ 2a+3b=4 \end{cases}$$

$31a=-4$ $\boxed{a=-\frac{4}{31}}$

$\boxed{b=\frac{44}{31}}$

2.) • Guessing the value of δ.

Let $\varepsilon>0$. We want to find $\delta>0$ such that $|(2x+1)-3|<\varepsilon$ whenever $0<|x-1|<\delta$.

We have $|(2x+1)-3| = |2x-2| = 2|x-1| < \varepsilon$

means $|x-1|<\varepsilon/2$

This suggest that we should choose $\delta=\varepsilon/2$.

• Prove that the value of δ works.

Let $\varepsilon>0$ be given. Let $\delta=\varepsilon/2$

$0<|x-1|<\frac{\varepsilon}{2} \Rightarrow 2|x-1|<\varepsilon$

$\Rightarrow |(2x+1)-3|<\varepsilon$

$\Rightarrow \lim_{x\to 1} 2x+1 = 3$

3°) $\lim_{x\to+\infty}\frac{\sqrt{x^2+1}}{x} = \lim_{x\to+\infty}\frac{x\sqrt{1+\frac{1}{x^2}}}{x} = 1$

$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty}\frac{-x\sqrt{1+\frac{1}{x^2}}}{x} = -1$

$\underline{y=1}$ and $\underline{y=-1}$ are the horizontal asymptotes of y.