

MATH 101.10 (112)

Quiz 3 (Sects. 2.7-3.4)

Duration: 20mn

Name: _____

ID number: _____

1.) (3pts) Let $f(x) = \sqrt[3]{x^2} + \frac{1}{\sqrt[4]{x}}$. Find $f''(1)$.

2.) (3pts) Let $y = \frac{(1-\cos x)^2}{(1-\sin x)^2}$. Find $\frac{dy}{dx} \Big|_{x=\frac{\pi}{6}}$.

3.) (4pts) Find an equation of the tangent line to the curve $y = \frac{2x+1}{(x+3)^2}$ at $(-2, -3)$.

$$\begin{aligned}
 1) \quad f(x) &= x^{2/3} + x^{-1/4} \\
 f'(x) &= \frac{2}{3}x^{-1/3} - \frac{1}{4}x^{-5/4} \\
 f''(x) &= -\frac{2}{9}x^{-4/3} + \frac{5}{16}x^{-9/4} \\
 f''(1) &= -\frac{2}{9} + \frac{5}{16} = \frac{13}{144}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \frac{dy}{dx} &= \frac{2(1-\cos x)(\sin x)(1-\sin x)^2 - 2(1-\sin x)(-\cos x)(1-\cos x)^2}{(1-\sin x)^4} \\
 &= \frac{2(1-\cos x)(\sin x)(1-\sin x) + 2\cos x(1-\cos x)^2}{(1-\sin x)^3} \\
 &= \frac{2(1-\cos x)[\sin x - \sin^2 x + \cos x - \cos^2 x]}{(1-\sin x)^3} \\
 &= \frac{2(1-\cos x)(\sin x + \cos x - 1)}{(1-\sin x)^3}
 \end{aligned}$$

$$\begin{aligned}
 \frac{dy}{dx} \Big|_{x=\frac{\pi}{6}} &= \frac{2(1-\cos \frac{\pi}{6})(\sin \frac{\pi}{6} + \cos \frac{\pi}{6} - 1)}{(1-\sin \frac{\pi}{6})^3} \\
 &= \frac{2(1-\frac{\sqrt{3}}{2})(\frac{1}{2} + \frac{\sqrt{3}}{2} - 1)}{(1-\frac{1}{2})^3} \\
 &= 4(-5 + 3\sqrt{3})
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \frac{dy}{dx} &= \frac{2(x+3)^2 - 2(x+3)(2x+1)}{(x+3)^4} \\
 &= \frac{2(x+3) - 2(2x+1)}{(x+3)^3} \\
 &= \frac{-2x+4}{(x+3)^3}
 \end{aligned}$$

$$\frac{dy}{dx} \Big|_{x=-2} = 8$$

An equation of the tangent line to y at $(-2, -3)$ is given by

$$y + 3 = 8(x + 2)$$

$$\boxed{y = 8x + 13}$$