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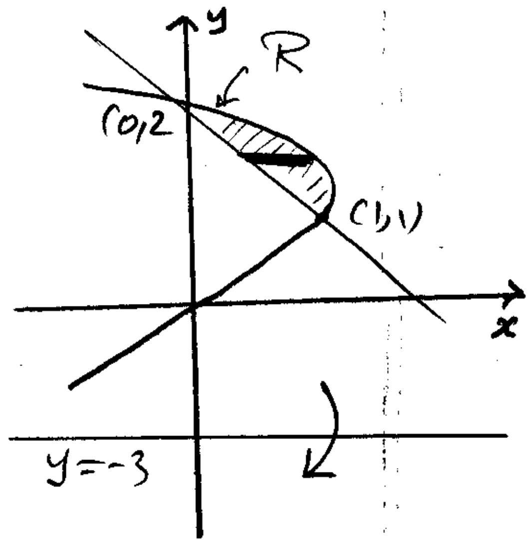
| KFUPM | Term 112 | Name Solutions | Serial_____ |
| MATH 202-12 | Quiz # 3 | ID # _____ | Section____ |

1) (5-points) Set up an integral, but do not evaluate, for the volume of the solid generated by rotating a region $R$ bounded by the curve $x = 2y - y^2$ and the line $x+y=2$ about the line $y=-3$. [Sketch R]




Points of intersection: $2y - y^2 = 2 - y$

$\Rightarrow y^2 - 3y + 2 = 0 = (y-1)(y-2) \Rightarrow$

$(1,1), (0,2)$

We use cylindrical shells:

$$\text{Volume} = \int_1^2 2\pi (y+3)\left[(2y-y^2) - (2-y)\right] dy$$

$$= \int_1^2 2\pi (y+3)(3y - y^2 - 2)\, dy$$

2) (5-Points) Find the average value of the function $f(x) = \frac{x}{(2x^2+1)^{3/2}}$ on the interval [0,2]

$$f_{ave} = \frac{1}{2-0} \int_0^2 \frac{x}{(2x^2+1)^{3/2}}\, dx$$

Let $u = 2x^2+1 \Rightarrow du = 4x\, dx \Rightarrow x\, dx = \frac{1}{4} du$

and $\boxed{x=0 \Rightarrow u=1}$ & $x=2 \Rightarrow \boxed{u=9} \Rightarrow$

$$f_{ave} = \frac{1}{2} \int_1^9 \frac{1}{u^{3/2}} \left(\frac{1}{4} du\right) = \frac{1}{8} \int_1^9 u^{-3/2}\, du$$

$\Longrightarrow$

you may continue on the back side $\longrightarrow$

$$f_{ave} = \frac{1}{8}\left[-2u^{-1/2}\right]_1^9 = -\frac{1}{4}\left[\frac{1}{3}-1\right] = -\frac{1}{4}\left(-\frac{2}{3}\right)$$

$$= \frac{1}{6}.$$

3) (5-Points) Evaluate $\int x^3 e^{-2x^2}\,dx$. $= \int x^2 \left(x e^{-2x^2} dx\right) = I$

We use integration by parts:

Let $u = x^2$, $dv = x e^{-2x^2} dx$

$\Rightarrow du = 2x\,dx$, $v = -\frac{1}{4} e^{-2x^2}$

$$\Rightarrow I = -\frac{1}{4}x^2 e^{-2x^2} + \frac{1}{2}\int x e^{-2x^2} dx$$

$$= -\frac{1}{4}x^2 e^{-2x^2} - \frac{1}{8} e^{-2x^2} + C.$$