KFUPM Term (112) Name Solutions Serial#________

MATH 102-12 Quiz # 6 ID#________ Section____

---

1) (4-Points) Use the comparison test to determine whether the series $\sum_{n=2}^{\infty} \frac{n^2}{\sqrt[3]{n^7-3n}}$ is convergent or divergent.

$$a_n = \frac{n^2}{\sqrt[3]{n^7-3n}} > b_n = \frac{n^2}{n^{7/3}} = \frac{1}{n^{1/3}}$$

But $\sum_{n=2}^{\infty} \frac{1}{n^{1/3}}$ is a divergent p-series

$\Rightarrow \sum_{n=2}^{\infty} a_n$ is divergent by the comparison test.

2) (4-Points) Use the limit comparison test to determine whether the series $\sum_{n=1}^{\infty}\left(5+\frac{1}{n}\right)^2 3^{-n}$ is convergent or divergent.

$a_n = \left(5+\frac{1}{n}\right)^2 3^{-n}$ and let $b_n = 3^{-n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty}\left(5+\frac{1}{n}\right)^2 = 25 > 0.$$

But $\sum_{n=1}^{\infty} 3^{-n} = \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$ is a convergent geometric series $\Rightarrow \sum_{n=1}^{\infty} a_n$ is convergent by the limit comparison test.

3) (4-Points) Determine whether, or not the alternating series test is applicable to the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{5^n}$

$b_n = \frac{n}{3^n}$. Let $f(x) = \frac{x}{3^x} \Rightarrow$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{3^x} \overset{H.R}{=} \lim_{x\to\infty} \frac{1}{3^x \ln 3} = 0$

and $f'(x) = \frac{3^x - x 3^x \ln 3}{3^{2x}} = \frac{3^x(1-x\ln 3)}{3^{2x}} < 0$

$\Rightarrow$ f is decreasing $\Rightarrow$ The series is convergent by the alternating series test.

4) (6-Points) Determine whether the series $1 - \frac{2^2+1}{2^3+1} + \frac{3^2+1}{3^3+1} - \frac{4^2+1}{4^3+1} + \cdots$ is absolutely convergent or conditionally convergent.

The series can be written as

$$\sum_{n=0}^{\infty} (-1)^n \frac{n^2+1}{n^3+1} = \sum_{n=0} a_n$$

$|a_n| = \frac{n^2+1}{n^3+1}$. Let $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{|a_n|}{b_n} = \lim_{n\to\infty} \frac{n^3+n}{n^3+1} = 1$$

$\Rightarrow \sum |a_n|$ is divergent by the limit comparism test

$\Rightarrow \sum a_n$ is not absolutely cgt.

$b_n = \frac{n^2+1}{n^3+1} \to 0$ as $n\to\infty$

and $b_{n+1} = \frac{(n+1)^2+1}{(n+1)^3+1} < b_n = \frac{n^2+1}{n^3+1}$

$\Rightarrow \sum_{n=0}^{\infty} (-1)^n b_n$ is cgt by the alternating series test $\Rightarrow$ the given series is conditionally cgt.