Q1. (a) Sketch the parametric curve **C:** $x = 2t - 1,\ y = t + t^2$. Indicate by an arrow how the graph is traced as t increases.

(b) Find an equation of the tangent line to the curve **C** at the point $(1, 2)$.

Q2. Find the surface area obtained by rotating the curve $x = \cos^2 t,\ y = \sin^2 t$, $0 \le t \le \pi/2$ about the y-axis.

Q3. (a) Find all values of c such that vectors $\vec{v} = \langle c, 5, 2\rangle$ and $\vec{w} = \langle 3c, c, -1\rangle$ are orthogonal

(b) Find the direction cosines of the vector $\vec{u} = \langle 2, 1, -2\rangle$

Q4. Find the area of the region that lies inside both the polar curves $r = 2 + 2\cos\theta$ and $r = 6\cos\theta$.

Q5. (a) Given the points $A(2, 1, -1),\ B(3, 0, 2),\ C(4, -2, -1)$ and $D(3, m, 0)$, find the volume of the parallelepiped with adjacent edges $AB,\ AC,\ AD$.

(b) Find all values of m such that the volume of the parallelepiped in (a) is 4.

Q1. If $\langle a, b, c\rangle$ is a **unit vector** orthogonal to both vectors $\langle 1, -1, 1\rangle$, $\langle 0, 4, 4\rangle$ then $|a| + |b| + |c|$ equals:

(A) $\dfrac{4}{\sqrt{6}}$

(B) $\dfrac{8}{\sqrt{6}}$

(C) $\dfrac{10}{\sqrt{6}}$

(D) $\dfrac{2}{\sqrt{6}}$

(E) $\dfrac{1}{\sqrt{6}}$

Q2. The **vector projection** ($\text{proj}_{\vec{a}}\vec{b}$) of $\vec{b} = 3\vec{i} - 7\vec{j} + 2\vec{k}$ onto $\vec{a} = \vec{i} - 2\vec{k}$ is

(A) $\dfrac{1}{5}\langle -1, 0, 2\rangle$

(B) $\dfrac{1}{\sqrt{5}}\langle -1, 0, 2\rangle$

(C) $\dfrac{1}{62}\langle -3, 7, -2\rangle$

(D) $\dfrac{1}{5}\langle -3, 7, -2\rangle$

(E) $\dfrac{1}{62}\langle -1, 0, 2\rangle$

Q3. The **slope** of the **tangent line** to the polar curve $r = \sec(3\theta)$ at $\theta = \pi/3$ is

(A) $-\dfrac{1}{\sqrt{3}}$

(B) $\dfrac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

(E) 0

Q4. The parametric curve $x = t - 2\ln t,\ y = t + \ln t$ is concave upward on the interval

(A) $(0, 2)$

(B) $(-\infty, 0)$

(C) $(0, \infty)$

(D) $(-\infty, 0) \cup (2, \infty)$

(E) $(-2, 0)$

Q5. A **Cartesian equation** of the polar curve $r = \csc\theta + \sec\theta$ is

(A) $xy = x + y$

(B) $xy = x - 1$

(C) $x^2y = x^2 + y$

(D) $xy = y^2 - 1$

(E) $xy^2 = x + y$

Q6. A vector $\vec{v}$ of magnitude 3 units that has the direction opposite to the vector $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ is

(A) $\frac{1}{\sqrt{14}}\langle -3, -6, 9\rangle$

(B) $\frac{1}{\sqrt{14}}\langle 3, 6, -9\rangle$

(C) $\frac{1}{\sqrt{14}}\langle -1, -2, 3\rangle$

(D) $\frac{1}{\sqrt{14}}\langle 1, 2, -3\rangle$

(E) $\frac{1}{\sqrt{14}}\langle 2, 4, -6\rangle$

Q7. The distance from the point $P\,(2,3,1)$ to the center of the sphere $3x^2 + 3y^2 + 3z^2 - 6x - 9y + 12z = 1$ is equal to

(A) $\frac{7}{2}$

(B) $\frac{5}{2}$

(C) $\frac{1}{2}$

(D) $\frac{9}{2}$

(E) $\frac{3}{2}$

Q1. A **Cartesian equation** of the polar curve $r = 2\csc\theta + \sec\theta$ is

(A) $xy = 2y^2 - 1$

(B) $xy = 2x - 1$

(C) $x^2y = 2x^2 + y$

(D) $xy = 2x + y$

(E) $xy^2 = 2x + y$

Q2. If $\langle a, b, c\rangle$ is a **unit vector** orthogonal to both vectors $\langle 1, -1, 1\rangle$, $\langle 0, 4, 4\rangle$, then $2|a| + |b| + |c|$ equals:

(A) $\dfrac{2}{\sqrt{6}}$

(B) $\sqrt{6}$

(C) $\dfrac{1}{\sqrt{6}}$

(D) $2\sqrt{6}$

(E) $\dfrac{3}{\sqrt{6}}$

Q3. A vector $\vec{v}$ of magnitude 3 units that has the direction opposite to the vector $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ is

(A) $\frac{1}{\sqrt{14}}\langle 2, 4, -6\rangle$

(B) $\frac{1}{\sqrt{14}}\langle 1, 2, -3\rangle$

(C) $\frac{1}{\sqrt{14}}\langle -3, -6, 9\rangle$

(D) $\frac{1}{\sqrt{14}}\langle 3, 6, -9\rangle$

(E) $\frac{1}{\sqrt{14}}\langle -1, -2, 3\rangle$

Q4. The distance from the point $P\,(2, 1, -3)$ to the center of the sphere $3x^2 + 3y^2 + 3z^2 - 6x - 9y + 12z = 1$ is equal to

(A) $\frac{9}{2}$

(B) $\frac{1}{2}$

(C) $\frac{5}{2}$

(D) $\frac{3}{2}$

(E) $\frac{7}{2}$

Q5. The **slope** of the **tangent line** to the polar curve $r = \sec(3\theta)$ at $\theta = \pi/3$ is

(A) $\sqrt{3}$

(B) $\dfrac{1}{\sqrt{3}}$

(C) 0

(D) $-\sqrt{3}$

(E) $-\dfrac{1}{\sqrt{3}}$

Q6. The **vector projection** $(\text{proj}_{\vec{a}}\vec{b})$ of $\vec{b} = -2\vec{i} - 6\vec{j} + \vec{k}$ onto $\vec{a} = \vec{i} - 2\vec{k}$ is

(A) $\dfrac{4}{5}\langle -1, 0, 2\rangle$

(B) $\dfrac{4}{41}\langle 2, 6, -1\rangle$

(C) $\dfrac{4}{\sqrt{5}}\langle -1, 0, 2\rangle$

(D) $\dfrac{4}{41}\langle 1, 0, -2\rangle$

(E) $\dfrac{4}{5}\langle 2, 6, -1\rangle$

Q7. The parametric curve $x = t - 3\ln t,\ y = t + \ln t$ is concave upward on the interval

(A) $(-\infty, 0) \cup (3, \infty)$

(B) $(3, \infty)$

(C) $(0, 3)$

(D) $(0, \infty)$

(E) $(-\infty, 0)$

Q1. The parametric curve $x = t - 3\ln t,\ y = t + \ln t$ is concave upward on the interval

(A) $(-\infty, 0)$

(B) $(-\infty, 0) \cup (3, \infty)$

(C) $(0, 3)$

(D) $(3, \infty)$

(E) $(0, \infty)$

Q2. The distance from the point $P\,(2, 3, 1)$ to the center of the sphere
$3x^2 + 3y^2 + 3z^2 - 6x - 9y + 12z = 1$ is equal to

(A) $\frac{5}{2}$

(B) $\frac{7}{2}$

(C) $\frac{3}{2}$

(D) $\frac{9}{2}$

(E) $\frac{1}{2}$

Q3. A vector $\vec{v}$ of magnitude 3 units that has the direction opposite to the vector $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ is

(A) $\frac{1}{\sqrt{14}}\langle -3, -6, 9\rangle$

(B) $\frac{1}{\sqrt{14}}\langle 2, 4, -6\rangle$

(C) $\frac{1}{\sqrt{14}}\langle -1, -2, 3\rangle$

(D) $\frac{1}{\sqrt{14}}\langle 1, 2, -3\rangle$

(E) $\frac{1}{\sqrt{14}}\langle 3, 6, -9\rangle$

Q4. If $\langle a, b, c\rangle$ is a **unit vector** orthogonal to both vectors $\langle 1, -1, 1\rangle$, $\langle 0, 4, 4\rangle$, then $|a| + 2|b| + |c|$ equals:

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{7}{\sqrt{6}}$

(C) $\frac{9}{\sqrt{6}}$

(D) $\frac{3}{\sqrt{6}}$

(E) $\frac{5}{\sqrt{6}}$

Q5. The **slope** of the **tangent line** to the polar curve $r = \sec(3\theta)$ at $\theta = \pi/3$ is

(A) $-\sqrt{3}$

(B) 0

(C) $-\dfrac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

(E) $\dfrac{1}{\sqrt{3}}$

Q6. The **vector projection** $(\text{proj}_{\vec{a}}\vec{b})$ of $\vec{b} = \vec{i} - 7\vec{j} + 2\vec{k}$ onto $\vec{a} = \vec{i} - 2\vec{k}$ is

(A) $\dfrac{3}{5}\langle -1, 0, 2\rangle$

(B) $\dfrac{1}{\sqrt{6}}\langle -1, 7, -2\rangle$

(C) $\dfrac{3}{\sqrt{5}}\langle -1, 0, 2\rangle$

(D) $\dfrac{3}{5}\langle -1, 7, -2\rangle$

(E) $\dfrac{1}{\sqrt{6}}\langle -1, 0, 2\rangle$

Q7. A **Cartesian equation** of the polar curve $r = \csc\theta + 2\sec\theta$ is

(A) $xy^2 = x + 2y$

(B) $x^2y = 2x^2 + y$

(C) $2xy = 2x - 1$

(D) $xy = x + 2y$

(E) $xy = 2y^2 - 1$

Q1. The **vector projection** ($\text{proj}_{\vec{a}}\vec{b}$) of $\vec{b} = 4\vec{i} - 6\vec{j} + 3\vec{k}$ onto $\vec{a} = \vec{i} - 2\vec{k}$ is

(A) $\frac{2}{\sqrt{5}}\langle -1, 0, 2\rangle$

(B) $\frac{2}{5}\langle -1, 0, 2\rangle$

(C) $\frac{2}{61}\langle -4, 6, -3\rangle$

(D) $\frac{2}{5}\langle -4, 6, -3\rangle$

(E) $\frac{2}{61}\langle -1, 0, 2\rangle$

Q2. A vector $\vec{v}$ of magnitude 3 units that has the direction opposite to the vector $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ is

(A) $\frac{1}{\sqrt{14}}\langle 3, 6, -9\rangle$

(B) $\frac{1}{\sqrt{14}}\langle 1, 2, -3\rangle$

(C) $\frac{1}{\sqrt{14}}\langle 2, 4, -6\rangle$

(D) $\frac{1}{\sqrt{14}}\langle -3, -6, 9\rangle$

(E) $\frac{1}{\sqrt{14}}\langle -1, -2, 3\rangle$

Q3. The **slope** of the **tangent line** to the polar curve $r = \sec(3\theta)$ at $\theta = \pi/3$ is

(A) $-\dfrac{1}{\sqrt{3}}$

(B) 0

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

(E) $\dfrac{1}{\sqrt{3}}$

Q4. A **Cartesian equation** of the polar curve $r = 3\csc\theta + \sec\theta$ is

(A) $x^2y = 3x^2 + y$

(B) $xy = 3x - 1$

(C) $xy^2 = x + 3y$

(D) $xy = 3x + y$

(E) $xy = 3y^2 - 1$

Q5. The distance from the point $P\,(2,1,-3)$ to the center of the sphere $3x^2+3y^2+3z^2-6x-9y+12z=1$ is equal to

(A) $\frac{7}{2}$

(B) $\frac{9}{2}$

(C) $\frac{5}{2}$

(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

Q6. The parametric curve $x=t-2\ln t,\ y=t+\ln t$ is concave upward on the interval

(A) $(-\infty,0)$

(B) $(0,2)$

(C) $(-\infty,0)\cup(2,\infty)$

(D) $(2,\infty)$

(E) $(0,\infty)$

Q7. If $\langle a, b, c\rangle$ is a **unit vector** orthogonal to both vectors $\langle 1, -1, 1\rangle$, $\langle 0, 4, 4\rangle$, then $3|a| + |b| + |c|$ equals:

(A) $\frac{5}{\sqrt{6}}$

(B) $\frac{4}{\sqrt{6}}$

(C) $\frac{8}{\sqrt{6}}$

(D) $\frac{2}{\sqrt{6}}$

(E) $\frac{1}{\sqrt{6}}$