MATH 202.15 (Term 112)

Quiz 3 (Sects. 4.1-4.3) Duration: 20mn

Name: **ID number:**

1.)(2pts)(2pts)Show that $\{e^{nx}, e^{-nx}\}$ is a fundamental set of solutions to the DE $y'' - n^2y = 0$.

2.)(4pts)Knowing that $y_1 = x\cos(\sqrt{2}\ln x)$ is a solution to the DE $-x^2y''+xy'-2y=0$, find a second solution y_2 linearly independent to y_1 by using reduction of order.

3.)(4pts)Find the genaral solution of the DE $y^{(4)} + 6y'' + 9y = 0$.

1-) It is clear that $y_1 = e^{nx}$ and $y_2 = e^{-nx}$ are solutions to the DE $y''-n^2y=0$ on $(-\infty,+\infty)$.

$$W = \begin{vmatrix} e^{nx} & e^{-nx} \\ ne^{nx} & -ne^{-nx} \end{vmatrix} = -n-n = -2n \neq 0$$

Thus, $\{e^{nx}, e^{-nx}\}$ is a fundamental set of solutions of $y''-n^2y=0$ on $(-\infty,+\infty)$.

2.) $$y_2(x) = y_1(x)\int \frac{e^{-\int P(x)\,dx}}{y_1^2(x)}\,dx$$

We write the DE in the standard form $y'' - \frac{1}{x}y' + \frac{2}{x^2}y = 0$, $x>0$

$P(x) = -\frac{1}{x}$

$$e^{-\int P(x)} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x, \quad x>0$$

So, $$y_2(x) = y_1(x)\int \frac{x}{x^2\cos^2(\sqrt{2}\ln x)}\,dx$$

$$= y_1(x)\int \frac{1}{x\cos^2(\sqrt{2}\ln x)}\,dx$$

using the substitution $u = \sqrt{2}\ln x$

$du = \sqrt{2}\,\frac{dx}{x}$

$$\int \frac{1}{x\cos^2(\sqrt{2}\ln x)}\,dx = \frac{1}{\sqrt{2}}\int \frac{du}{\cos^2 u} = \frac{1}{\sqrt{2}}\tan(u)$$

$$= \frac{1}{\sqrt{2}}\tan(\sqrt{2}\ln x)$$

So, that

$$y_2(x) = \frac{1}{\sqrt{2}}x\cos(\sqrt{2}\ln x)\tan(\sqrt{2}\ln x)$$

$$\boxed{y_2(x) = \frac{1}{\sqrt{2}}x\sin(\sqrt{2}\ln x)}$$

or $\boxed{y_2(x) = x\sin(\sqrt{2}\ln x)}$

3.) The auxiliary equation is

$m^4 + 6m^2 + 9 = 0$

$(m^2+3)^2 = 0$

$m = \pm\sqrt{3}i$ | roots of multiplicity 2

The general solution of the DE is

$$y = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x + c_3x\cos\sqrt{3}x + c_4x\sin\sqrt{3}x, \quad x\in(-\infty,+\infty)$$

where c_1, c_2, c_3, c_4 are arbitrary constants.

MATH 202.18 (Term 112)

Quiz 3 (Sects. 4.1-4.3) Duration: 20mn

Name: **ID number:**

1.)(2pts)Show that $\{\cos nx, \sin nx\}$ is a fundamental set of solutions to the DE $y'' + n^2y = 0$.

2.)(4pts)Knowing that $y_1 = x^{\frac{1}{4}}\cos(\frac{\sqrt{7}}{4}\ln x)$ is a solution to the DE $2x^2y'' + xy' + y = 0$, find a second solution y_2 linearly independent to y_1 by using reduction of order.

3.)(4pts)Find the general solution of the DE $y''' + y'' - y' - y = 0$.

1.) It is clear that $y_1 = \cos nx$ and $y_2 = \sin nx$ are solutions of the DE $y'' + n^2 y = 0$ on $(-\infty, +\infty)$.

$$W = \begin{vmatrix} \cos nx, & \sin nx \\ -n\sin nx & n\cos nx \end{vmatrix} = n\cos^2 nx + n\sin^2 nx = n \neq 0$$

thus, $\{\cos nx, \sin nx\}$ form a fundamental set of solutions of the DE on $(-\infty, +\infty)$.

2.)
$$y_2(x) = y_1(x)\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}\,dx$$

We write the DE in its standard form $y'' + \frac{1}{2x}y' + \frac{1}{2x^2}y = 0$, $x>0$

$P(x) = \frac{1}{2x}$

$$e^{-\int P(x)dx} = e^{-\int \frac{1}{2x}dx} = e^{-\frac{1}{2}\ln x} = x^{-\frac{1}{2}},\ x>0$$

$$y_2(x) = y_1(x)\int \frac{x^{-1/2}}{x^{\frac{1}{2}}\cos^2(\frac{\sqrt{7}}{4}\ln x)}\,dx$$

$$= y_1(x)\int \frac{1}{x\cos^2(\frac{\sqrt{7}}{4}\ln x)}\,dx.$$

Using the substitution $u = \frac{\sqrt{7}}{4}\ln x$.

$$du = \frac{\sqrt{7}}{4}\,\frac{dx}{x}$$

$$\Rightarrow \int \frac{1}{x\cos^2(\frac{\sqrt{7}}{4}\ln x)}\,dx = \frac{4}{\sqrt{7}}\int \frac{du}{\cos^2 u} = \frac{4}{\sqrt{7}}\tan u$$

$$= \frac{4}{\sqrt{7}}\tan\left(\frac{\sqrt{7}}{4}\ln x\right)$$

So that, $y_2(x) = \frac{4}{\sqrt{7}}x^{\frac{1}{4}}\cos(\frac{\sqrt{7}}{4}\ln x)\tan(\frac{\sqrt{7}}{4}\ln x)$

$$\boxed{y_2(x) = \frac{4}{\sqrt{7}}x^{1/4}\sin\left(\frac{\sqrt{7}}{4}\ln x\right)}$$

or

$$\boxed{y_2 = x^{1/4}\sin\left(\frac{\sqrt{7}}{4}\ln x\right)}$$

3.) The auxiliary equation is

$$m^3 + m^2 - m - 1 = 0$$

$$(m-1)(m+1)^2 = 0$$

$m = 1$ and $m = -1$ (multiplicity

The general solution of the DE is

$$y = C_1e^{x} + C_2e^{-x} + C_3xe^{-x},\quad x\in(-\infty, +\infty)$$

where C_1, C_2, C_3 are arbitrary constants.