Prob. 1
Consider the system
\[
\begin{align*}
y_1' &= y_2^2 \\
y_2' &= y_1 + y_2
\end{align*}
\]
(a) Check that \( \phi_1(t) = \frac{\eta_1}{1-\eta_1(t-t_0)} \), \( \phi_2(t) = \eta_2 e^{t-t_0} + \eta_1 \int_{t_0}^{t} e^{s-t_0} ds \) is a solution for which \( \phi_1(t_0) = \eta_1 \), \( \phi_2(t_0) = \eta_2 \).
(b) Discuss the interval of existence according to \( \eta_1 > 0 \), \( \eta_1 = 0 \), \( \eta_1 < 0 \).

Prob. 2
Consider the D.E.
\[
y' = \begin{cases} 
0, & t \leq 0, \quad -\infty < y < \infty \\
2\sqrt{y}, & t \geq 0, \quad 0 \leq y < \infty \\
y^2, & t \geq 0, \quad -\infty < y < 0
\end{cases}
\]
Determine whether \( \phi(t) = \begin{cases} 
1, & t < 0 \\
(t+1)^2, & t \geq 0
\end{cases} \) is a solution on \( -\infty < t < \infty \).

Prob. 3
Show that \( \phi(t) = -1/t \) is a solution of \( y' = y^2 \) passing through \((-1, 1)\) and it is the only solution passing through \((-1, 1)\). What is the largest interval on which it is a solution. What is your conclusion?

Prob. 4
Discuss the existence and uniqueness of solutions $\phi$ of $y'' + p(t)y' + q(t)y = f(t)$, $\phi(t_0) = y_0$, $\phi'(t_0) = z_0$.

**Prob. 5**

(a) Show that $\phi(t) \equiv 0$ is the only solution of $y'' + p(t)y' + q(t)y = 0$, $\phi(0) = \phi'(0) = 0$, if $p$ and $q$ are continuous on some interval containing 0 in its interior.

(b) Show that if $\psi(t)$ is a solution of the D.E. $y'' + p(t)y' + q(t)y = 0$ that is tangent to the $t$-axis at some point $(t_1, 0, 0)$ then $\psi(t) \equiv 0$.

**Prob. 6**

Find all continuous functions which are nonnegative on $0 \leq t \leq 1$ such that $f(t) \leq \int_0^t f(s)ds$, $0 \leq t \leq 1$. 