Let $R_l$ be the set of real numbers with the lower limit topology, $R_u$ the set of real numbers with the usual topology, and $R_d$ is the set of real numbers with the discrete topology.

1. Show that $R \times R$ with the dictionary order topology is homeomorphic to $R_l \times R_u$.
2. Describe the subspace topology on a line $L$ in the plane topologized as $R_l \times R_u$ and also as $R_l \times R_l$.
3. Let $A \subseteq X$, $f : A \to Y$ be a continuous function and $Y$ be Hausdorff space. Show that if $f$ may be extended to a continuous function $g : A \to Y$ then $g$ unique.
4. Show that $R \times R$ in the dictionary order topology is matrizable.
5. Let $\mathbb{R}^\omega$ denotes the subset of $\mathbb{R}^\omega$ (countable product) consisting of sequences $(x_1, x_2, ..., x_n, ...)$ which are eventually zero i.e., $x_i \neq 0$ for at most finitely many $i$. What is $\overline{\mathbb{R}}^\omega$ in the box and product topologies on $\mathbb{R}^\omega$?
6. Consider $\mathbb{R}^\omega$ with the box and product topologies.

(a) In which topology (topologies) are $f, g, h : R \to \mathbb{R}^\omega$ continuous?

$$f(t) = (t, 2t, 3t, ...)$$
$$g(t) = (t, t, t, ...)$$
$$h(t) = \left( t, \frac{1}{2}t, \frac{1}{3}t, ... \right)$$

(b) In which do the following sequences converge?

$$w_1 = (1, 1, 1, 1, ...)$$
$$x_1 = (1, 1, 1, 1, ...)$$
$$w_2 = (0, 2, 2, 2, ...)$$
$$x_2 = \left( 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ... \right)$$
$$w_3 = (0, 0, 3, 3, ...)$$
$$x_3 = \left( 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ... \right)$$
$$\vdots$$
$$\langle w_n \rangle$$
$$\langle w_n \rangle$$
\[ y_1 = (1, 0, 0, 0, \ldots) \quad z_1 = (1, 1, 0, 0, \ldots) \]
\[ y_2 = \left( \frac{1}{2}, \frac{1}{2}, 0, 0, \ldots \right) \quad z_2 = \left( \frac{1}{2}, \frac{1}{2}, 0, 0, \ldots \right) \]
\[ y_3 = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots \right) \quad z_3 = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots \right) \]
\[ \vdots \]
\[ \langle y_n \rangle \quad \langle z_n \rangle \]

7. Let \( f_n : [0,1] \rightarrow \mathbb{R} \) be defined by \( f_n(x) = x^n \) for each positive integer \( n \). Show that \( \langle f_n(x) \rangle \) converges for each \( x \) in \([0,1]\) but \( \langle f_n \rangle \) does not converge uniformly.

8. Let \( X \) be a topological space and \( Y \) be a metric space. Let \( \{ f_n \} : X \rightarrow Y \) be a sequence of continuous functions and let \( \langle f_n \rangle \rightarrow f \) in \( X \). If \( \langle f_n \rangle \rightarrow f \) uniformly, show that \( \langle f_n(x) \rangle \rightarrow f(x) \).

9. (a) Is \( R^1 \) connected?
   (c) Show that \( R^w \) is not connected in the box topology.
   (d) Show that \( R^n \) and \( R \) are not homeomorphic if \( n > 1 \).

10. Cantor's middle third set \( C \).

\[ A_0 = [0,1], \quad A_1 = A_0 - \left( \frac{1}{2}, \frac{2}{3} \right), \quad A_2 = A_1 - \left( \frac{1}{9}, \frac{2}{9} \right) \cup \left( \frac{7}{9}, \frac{8}{9} \right) \]
\[ A_n = A_{n-1} - \bigcup_{k=0}^{n-1} \left( \frac{1+3^k}{3^n}, \frac{2+3^k}{3^n} \right) \]

\[ C = \bigcap_{n=1}^{\infty} A_n \] is called "the" Cantor set (which is a subspace of \([0,1]\)).

(a) \( C \) is totally disconnected, i.e., a point is the largest connected subset.
(b) \( C \) is compact.
(c) \( A_n \) is the union of finitely many closed intervals of length \( \frac{1}{3^n} \). Show also that their endpoints are also in \( C \).
(d) Show that every point of \( C \) is a limit point of \( C \).
(e) \( C \) is uncountable.