Name_________________________________________________________ ID#:___________ Serial #:_____

Instructions.
1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Financial calculators, mobile calculators, or communicable devices are disallowed. Use regular scientific calculator only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

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1. (2+2+2+2+2=10 marks) A principal of $100 accumulates to $140 over 3 years. Find the

a) effective annual interest rate

b) simple annual interest rate

c) nominal annual interest rate compounded quarterly

d) nominal annual discount rate compounded semiannually

e) force of interest.
2. (6+3=9 marks) Bader borrows $4,000 on January 1, 2010. He plans to repay this loan by a payment of \( X \) on September 1, 2010, another payment of \( X \) on January 1, 2011, and a last payment of $1,000 on July 1, 2011. You are given that \( i^{(5)} = 0.05 \) in 2010, and \( d^{(2)} = X/10000 \) in 2011.

a) Find \( X \).

b) Find the average effective annual interest rate for the two years.

3. (4+4=8 marks) A loan of $30,000 will be paid off by 15 level quarterly payments of \( P \), with interest at \( i^{(4)} = 8\% \).

a) If the first payment is 3 months after the loan is made, determine \( P \).

b) If the first payment is 9 months after the loan is made, determine \( P \).
4. (5+3=8 marks) The present value of a series of 40 payments starting at 200 at the end of the first year and increasing by 1 each year thereafter is equal to \( X \). The annual effective rate of interest is 6%.

a) Calculate \( X \).

b) Find the accumulated value of the series of payment immediately after the 40th payment.

5. (5+4=9 marks) At time \( t = 0 \), Samir deposits \( X \) into a fund which pays at an interest rate of 12% convertible quarterly while Jamal deposits \( X \) into a fund which pays at a force of interest of \( \delta_t = \frac{1}{1+t} \).

At time \( t = 6 \), the accumulated amount in Samir’s fund is 400 and the accumulated amount in Jamal’s fund is \( Y \).

a) Determine \( Y \).

b) At what time, \( t \), is the accumulated amounts from Jamal’s fund is four times its initial investment?
6. (6 marks) Kamal buys a perpetuity-due paying 60 annually which payments she deposits into a savings account earning interest at an annual effective rate of 5%. Twenty years later, before receiving the 21st payment, Kamal sells the perpetuity based on an effective annual interest rate of 5%. Using the proceeds from the sales plus the money in the savings account, Kamal purchases an annuity-due paying $X$ per year for 10 years at an effective annual rate of 5%. Calculate $X$. 

END OF TEST PAPER
**FORMULA SHEET**

Accumulated value of n-payment **annuity-immediate** of 1: \( s_{n|i} = \frac{(1 + i)^n - 1}{i} \)

Present value of n-payment annuity-immediate of 1: \( a_{n|i} = \frac{1 - v^n}{i} \)

Present value of a **perpetuity**-immediate: \( a_{\infty|i} = \frac{1}{i} \)

**Annuity**-due: \( \bar{s}_{n|i} = \frac{(1 + i)^n - 1}{d} \), \( \bar{a}_{n|i} = \frac{1 - v^n}{d} \)

Continuous annuities: \( \bar{s}_{n|i} = \int_0^n (1 + i)^{-t} dt = \frac{(1 + i)^n - 1}{\delta} \), \( \bar{a}_{n|i} = \int_0^n v^t dt = \frac{1 - v^n}{\delta} = \frac{1 - e^{-n\delta}}{\delta} \)

Present value of n-term **m**thly **payable** annuity-immediate of 1/m: \( a_{n|m}^{(m)} = \frac{1 - v^n}{\bar{i}^{(m)}} = a_{n|i} \times \frac{i}{\bar{i}} \)

Present value of annuity with **non-level** payments: \( K_1v + K_2v^2 + \cdots + K_{n-1}v^{n-1} + K_nv^n \)

Present value of annuity with payments following **geometric series**:

\[
v + (1 + r)v^2 + \cdots + (1 + r)^{n-2}v^{n-1} + (1 + r)^{n-1}v^n = \frac{1 - (1 + r)^n}{i - r}
\]

if \( i = r \), then \( v + (1 + r)v^2 + \cdots + (1 + r)^{n-2}v^{n-1} + (1 + r)^{n-1}v^n = v + v + \cdots + v = nv \)

**Accumulated** value of annuity with payments following **geometric series**:

\[
\frac{1 - (1 + r)^n}{i - r} (1 + i)^n = \frac{(1 + i)^n - (1 + r)^n}{i - r}
\]

Dividend discount model for **present value of a stock**: \( \frac{K}{i - r} \)

n-payment **increasing** annuity-immediate: \( (Is)_{n|i} = \frac{s_{n|i} - n}{i} \), \( (Ia)_{n|i} = \frac{a_{n|i} - n\bar{a}^n}{i} \)

n-payment **increasing perpetuity**-immediate: \( (Ia)_{\infty|i} = \frac{a_{\infty|i}}{i} = \frac{1}{i} = \frac{1}{i + 1} \)

n-payment **decreasing** annuity-immediate: \( (Ds)_{n|i} = \frac{n(1 + i)^n - s_{n|i}}{i} \), \( (Da)_{n|i} = \frac{n - a_{n|i}}{i} \)

**Sinking Fund**: (a) \( P = \frac{K \times s_{n|j}}{1 + i \times s_{n|j}} = (K - P \times i) s_{n|j} \) \( j \) (b) periodic Outlay: \( K = P \left[ i + \frac{1}{s_{n|j}} \right] \)

Capitalized Cost: \( C = P + \frac{P - S}{i \times s_{n|i}} + \frac{M}{i} \)

Periodic Charge = (Capitalized Cost) \( \times i = \left( P + \frac{P - S}{i \times s_{n|i}} + \frac{M}{i} \right) \times i = Pi + \frac{P - S}{s_{n|i}} + M \)

**Depreciation Methods**

1) **Declining Balance** (compound discount) Method: \( P_t = P_0 \times (1 - d)^t \) \( D_t = P_0 \times (1 - d)^{t-1} \cdot d \)

2) **Depreciation - Straight Line** Method: \( P_t = P_0 - t \times \frac{1}{n} \times (P_0 - P_n) \) \( D_t = \frac{1}{n} (P_0 - P_n) \)

3) **Depreciation - Sum of Year Digits** Method: \( S_k = 1 + 2 + \cdots + k = \frac{k(k+1)}{2} \)

\( P_t = P_n + S_{n-t} \times (P_0 - P_n) \) \( D_t = \frac{n - t + 1}{S_n} \times (P_0 - P_n) \)

4) **Depreciation - Compound interest** Method: \( P_t = P_0 - S_{n-t} \times (P_0 - P_n) \) \( D_t = \frac{(1 + i)^{t-1}}{S_n} \times (P_0 - P_n) \)

\[
i_{t+1} = \frac{A(t+1) - A(t)}{A(t)} , \quad A(t) = A(0)a(t) \]

accumulation factor: (a) Compound interest \( a(t) = (1 + i)^t \) \( (b) \) Simple interest \( a(t) = 1 + it \)

\[
v = \frac{1}{1 + i} , \quad 1 + i = \left[ 1 + \frac{i(m)}{m} \right]^m , \quad \frac{i(m)}{m} = m \left( (1 + i)^{1/m} - 1 \right) , \quad i(\infty) = \ln(1 + i)
\]

\[
d = \frac{A(1) - A(0)}{A(1)} = \frac{i}{1 + i} , \quad i = \frac{d}{1 - d} , \quad 1 - d = \left[ 1 - \frac{d(m)}{m} \right]^m , \quad d(\infty) = -\ln(1 - d)
\]

discount factor: (a) Compound discount factor \( (1 - d)^t \) \( (b) \) Simple discount factor \( 1 - dt \)

\[
\delta_t = \frac{A(t)}{A(t)} , \quad A(n) = A(0)e^{\int_0^n \delta dt} , \quad i_{real} = \frac{i - r}{1 + r}
\]

\[
1 + x + x^2 + x^3 + \cdots + x^k = \frac{1 - x^{k+1}}{1 - x} = \frac{x^{k+1} - 1}{x - 1}
\]