Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS201: Financial Mathematics
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Major Exam 2 FORM B Solution
November 27 2012
6.30pm-8.00pm

Name________________________________ ID#:__________ Serial #:___

Instructions.
1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two persons can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on. Return to it after you attempted other questions.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the extremely rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, or communicable devices are disallowed. Use scientific calculator with mathematical equation solving capability or SOA approved financial calculators only. No other materials such as lecture notes, assignments, solution, etc are allowed.
9. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Marks	Comments
1	10	
2	10	
3	10	
4	5	
5	5	
Total	40	

Extra blank page

1. (3+3+2+2 =10 marks) Today you borrow a loan of \$125,000 in order to repay it in 20 years at an interest rate of $i^{(12)} = 12\%$, compounded monthly. You amortize this loan with *monthly level payments* (meaning paying every month the same amount) starting one month from today.

a) Determine your monthly level payment.

b) What is your outstanding balance just after your payment 8 years from now?

c) How much principal do you repay in this payment 8 years from now?

d) What are the total interest payments over the 20 years?

2. (4+3+3=10 marks) Today you borrow a loan of \$125,000 in order to repay it in 20 years at an interest rate of $i^{(12)} = 12\%$, compounded monthly. For this loan, you deliver periodic payments only for interest, and a single payment of the principal of \$125,000 in 20 years. You amortize the loan with level payments into a separate sinking-fund, which earns an interest rate of $j^{(12)} = 9\%$, compounded monthly.

a) Find your monthly outlay (interest on original loan + deposit into sinking fund).

b) What is your outstanding balance just after your payment 8 years from now?

c) How much principal do you repay in this payment 8 years from now?

3. (5+5=10 marks) A bond with face value of \$2,500 matures on September 1, 2018.
   The semi-annual coupon rate is quoted at a nominal rate of $r^{(2)} = 7\%$

a) Determine the purchase price on March 1, 2007, which guarantees the buyer a yield of $j^{(2)} = 6.8\%$.

b) What is the market quotation or clean price, on Nov 18, 2010, which guarantees the buyer a yield of $j^{(2)} = 7.2\%$.

4. (5 marks) On January 1, 2009, an investment account is worth 100,000. On April 1, 2009, the value has increased to 103,000 and 9,000 is withdrawn. On January 1, 2011, the account is worth 104,000. Assuming a **dollar-weighted** method for 2009 and a **time-weighted** method for 2010, and the effective annual interest rate was equal to x for both 2009 and 2010. Calculate x.

5. (5 marks) A 30-year bond with a face value of 1000 and 12% coupons payable **quarterly** is selling at 800. Calculate the annual nominal yield rate convertible **quarterly**.

END OF TEST PAPER

FORMULA SHEET

IRR: Solve for j in $\sum_{k=0}^{n} C_k v_j^{t_k} = 0$
Profitability Index $I = \frac{\text{Present value of cash } \textbf{inflows}}{\text{Present value of cash } \textbf{outflows}}$
Payback Period = first k in $-\sum_{s=0}^{t} C_s \leq \sum_{r=t+1}^{k} C_r$
Discounted Payback Period = first k in $-\sum_{s=0}^{t} C_s v_i^s \leq \sum_{r=t+1}^{k} C_r v_i^r$
Dollar-weighted $I = B - [A + \sum_{k=1}^{n} C_k]$ $\qquad i = \frac{I}{A + \sum_{k=1}^{n} C_k(1-t_k)}$
Time-weighted $i = [\frac{F_1}{A} \times \frac{F_2}{F_1+C_1} \times \frac{F_3}{F_2+C_2} \times ... \times \frac{F_k}{F_{k-1}+C_{k-1}} \times \frac{B}{F_k+C_k}] - 1$
Trapezoidal rule to approx $\int_a^b f(x)dx \approxeq \frac{b-a}{2}[f(a)+f(b)]$
Descartes' rule of signs of Polynomial $P(x)$ for counting types of roots:
(i) $n_{+\text{ve roots}} \leq n_{\text{sign changes in } (C_n, C_{n-1}, ..., C_1, C_0)}$
(ii) $n_{-\text{ve roots}} \leq n_{\text{sign changes in } ((-1)^n C_n, (-1)^{n-1} C_{n-1}, ..., (-1)C_1, C_0)}$

$P = Cv_j^n + Fr \cdot a_{\overline{n|}j} = C + (Fr - Cj) \cdot a_{\overline{n|}j} = K + \frac{g}{j}(C-K)$ where $K = Cv_j^n$
$P = Fv_j^n + Fr \cdot a_{\overline{n|}j} = F + F(r-j) \cdot a_{\overline{n|}j} = K + \frac{r}{j}(F-K)$ where $F = C$
(i) $P = F$ Bought at Par (ii) $P > F$ Bought at a Premium (iii) $P < F$ Bought at a Discount
(i) $P = F \leftrightarrow r = j$ $\qquad$ (ii) $P > F \leftrightarrow r > j$ $\qquad$ (iii) $P < F \leftrightarrow r < j$
$t = \frac{\#\text{ of days since last coupon paid}}{\#\text{ of days in the coupon period}}$ $\qquad P_t = P_0(1+j)^t$ $\qquad price_t = P_t - tFr.$
$BV_{t+1} = BV_t(1+j) - Fr$ $\qquad I_{t+1} = BV_t \times j$ $\qquad PR_{t+1} = Fr - I_{t+1}$
$BV_t = F[1 + (r-j).a_{\overline{n-t|}}]$ $\qquad I_t = F[j + (r-j)(1 - v_j^{n-k+1})]$ $\qquad PR_t = F(r-j)v_j^{n-k+1}$

$L = K_1 v + K_2 v^2 + ... + K_n v^n$
$OB_{t+1} = OB_t(1+i) - K_{t+1}$ $\qquad I_{t+1} = OB_t \times i$ $\qquad PR_{t+1} = K_{t=1} - I_{t+1}$
Retrospective: $OB_t = OB_0(1+i)^t - K_1(1+i)^{t-1} - K_2(1+i)^{t-2} - ....K_{t-1}(1+i) - K_t$
Prospective: $OB_t = K_{t+1} \times v + K_{t+2} \times v^2 + ... + K_n \times v^{n-t}$
Level payments: $OB_t = L(1+i)^t - Ks_{\overline{t|}i} = K(a_{\overline{n|}i}(1+i)^t - s_{\overline{t|}i}) = K(a_{\overline{n-t|}})$
$I_t = Kia_{\overline{n-t+1|}} = K(1 - v^{n-t+1})$ $\qquad PR_t = Kv^{n-t+1}$ $\qquad PR_t = PR_{t-1}(1+i) = PR_1(1+i)^{t-1}$
Sinking Fund periodic Outlay: $L\left[i + \frac{1}{s_{\overline{n|}j}}\right] = I_t + PR_t$
Sinking Fund periodic Amortization schedule:
$OB_t = L\left[1 - \frac{s_{\overline{t|}j}}{s_{\overline{n|}j}}\right]$ $\qquad PR_t = OB_{t-1} - OB_t = L\frac{(1+j)^{t-1}}{s_{\overline{n|}j}}$ $\qquad I_t = L \cdot i - L\frac{s_{\overline{t-1|}j}}{s_{\overline{n|}j}} \times j = L\left[i - \frac{(1+j)^{t-1}-1}{s_{\overline{n|}j}}\right]$

Makeham's single loan: $A = Lv_j^n + L_s i a_{\overline{n|}j} = K + \frac{i}{j}(L-K)$
Makeham's m loans with scheduled repayments:
$A_s = L_s v_j^{t_s} + L_s i a_{\overline{t_s|}j} = K_s + \frac{i}{j}(L_s - K_s)$ $\qquad A = \sum_{s=1}^{m} A_s = \sum_{s=1}^{m}\left[K_s + \frac{i}{j}(L_s - K_s)\right] = K + \frac{i}{j}(L-K)$

Accumulated value of n-payment annuity-immediate of 1: $s_{\overline{n|}i} = \frac{(1+i)^n - 1}{i}$
Present value of n-payment annuity-immediate of 1: $a_{\overline{n|}i} = \frac{1-v^n}{i}$
Present value of a perpetuity-immediate: $a_{\overline{\infty|}i} = \frac{1}{i}$
Annuity-due: $\qquad \ddot{s}_{\overline{n|}i} = \frac{(1+i)^n-1}{d}$, $\qquad \ddot{a}_{\overline{n|}i} = \frac{1-v^n}{d}$
Continuous annuities: $\bar{s}_{\overline{n|}i} = \int_0^n (1+i)^{n-t}dt = \frac{(1+i)^n-1}{\delta}$, $\qquad \bar{a}_{\overline{n|}i} = \int_0^n v^t dt = \frac{1-v^n}{\delta} = \frac{1-e^{-n\delta}}{\delta}$
Present value of n-term m^{thly} payable annuity-immediate of $1/m$: $a_{\overline{n|}i}^{(m)} = \frac{1-v_i^n}{i^{(m)}} = a_{\overline{n|}i} \times \frac{i}{i^{(m)}}$

Present value of annuity with non-level payments: $K_1 v + K_2 v^2 + \cdots + K_{n-1}v^{n-1} + K_n v^n$
Present value of annuity with payments following geometric series:

$$v + (1+r)v^2 + \cdots + (1+r)^{n-2}v^{n-1} + (1+r)^{n-1}v^n = \frac{1 - (\frac{1+r}{1+i})^n}{i-r}$$

a) if $i = r$, then $v + (1+r)v^2 + \cdots + (1+r)^{n-2}v^{n-1} + (1+r)^{n-1}v^n = v + v + \cdots + v = nv$

Accumulated value of annuity with payments following geometric series: $\frac{1-(\frac{1+r}{1+i})^n}{i-r}(1+i)^n = \frac{(1+i)^n-(1+r)^n}{i-r}$
Dividend discount model for present value of a stock: $\frac{K}{i-r}$
n-payment increasing annuity-immediate: $(Is)_{\overline{n}|} = \frac{\ddot{s}_{\overline{n}|}-n}{i}$, $(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|}-nv^n}{i}$
n-payment increasing perpetuity-immediate: $(Ia)_{\overline{\infty}|} = \frac{\ddot{a}_{\overline{\infty}|}}{i} = \frac{1}{id} = \frac{1}{i} + \frac{1}{i^2}$
n-payment decreasing annuity-immediate: $(Ds)_{\overline{n}|i} = \frac{n(1+i)^n - s_{\overline{n}|}}{i}$, $(Da)_{\overline{n}|} = \frac{n-a_{\overline{n}|}}{i}$
Capitalized Cost: $C = P + \frac{P-S}{i \times S_{\overline{n}|i}} + \frac{M}{i}$
Periodic Charge = (Capitalized Cost) $\times i = \left(P + \frac{P-S}{i \times S_{\overline{n}|i}} + \frac{M}{i}\right) \times i = Pi + \frac{P-S}{S_{\overline{n}|i}} + M$
Depreciation Methods
1) Declining Balance (compound discount) Method: $P_t = P_0 \times (1-d)^t$ $D_t = P_0 \times (1-d)^{t-1} \cdot d$
2) Depreciation - Straight Line Method: $P_t = P_0 - t \times \frac{1}{n} \times (P_0 - P_n)$ $D_t = \frac{1}{n}(P_0 - P_n)$
3) Depreciation - Sum of Year Digits Method: $S_k = 1 + 2 + \cdots + k = \frac{k(k+1)}{2}$
$P_t = P_n + \frac{S_{n-t}}{S_n} \times (P_0 - P_n)$ $D_t = \frac{n-t+1}{S_n} \times (P_0 - P_n)$
4) Depreciation - Compound interest Method: $P_t = P_0 - \frac{S_{\overline{t}|i}}{S_{\overline{n}|i}} \times (P_0 - P_n)$ $D_t = \frac{(1+i)^{t-1}}{S_{\overline{n}|i}} \times (P_0 - P_n)$

$i_{t+1} = \frac{A(t+1)-A(t)}{A(t)}$, $A(t) = A(0)a(t)$

a) $a(t) = (1+i)^t$ Compound interest accumulation factor

b) $a(t) = 1 + it$ Simple interest accumulation factor

$v = \frac{1}{1+i}$, $1 + i = \left[1 + \frac{i^{(m)}}{m}\right]^m$ $i^{(m)} = m\left[(1+i)^{1/m} - 1\right]$ $i^{(\infty)} = ln(1+i)$

$d = \frac{A(1)-A(0)}{A(1)} = \frac{i}{1+i}$ $i = \frac{d}{1-d}$

a) $(1-d)^t$ Compound discount factor

b) $1 - dt$ Simple discount factor

$1 - d = \left[1 - \frac{d^{(m)}}{m}\right]^m$ $d^{(\infty)} = -ln(1-d)$

$\delta_t = \frac{A'(t)}{A(t)}$ $A(n) = A(0)e^{\int_0^n \delta_t dt}$, $i_{real} = \frac{i-r}{1+r}$

$1 + x + x^2 + x^3 + \cdots + x^k = \frac{1-x^{k+1}}{1-x} = \frac{x^{k+1}-1}{x-1}$