

1. The sum of all values of k for which $y = k$ is a horizontal asymptote to the graph of the function

$$f(x) = 2 + \left(\frac{8x^4 + x^2 - 1}{27x^4 + 8} \right)^{1/3} \text{ is}$$

(a) $\frac{8}{3}$

(b) 2

(c) $\frac{62}{27}$

(d) $\frac{5}{2}$

(e) 0

2. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 3x - 3x - x^2}{1 - \cos 2x}$.

(a) $-\frac{1}{2}$

(b) $\frac{2}{3}$

(c) -2

(d) $-\frac{2}{3}$

(e) Does not exist

3. If the radius of a circle is increased from 3 m to 3.06 m, then the estimated percentage change in the area of the circle is

- (a) 4%
- (b) 6%
- (c) 9%
- (d) 2%
- (e) 0.36%

4. Using the midpoint rule and 3 rectangles of equal base, the estimated area under the curve $y = \sin(\pi x)$ between $x = 0$ and $x = 1$ is equal to

- (a) $\frac{2}{3}$
- (b) $\frac{1}{3}(1 + \sqrt{3})$
- (c) $\frac{1}{\sqrt{3}}$
- (d) $\frac{2}{\pi}$
- (e) 1

5. $\int \left(4 \sin 2x - \frac{9}{e^{3x}} \right) dx =$

(a) $-2 \cos 2x + \frac{3}{e^{3x}} + C$

(b) $-8 \cos 2x - \frac{27}{e^{3x}} + C$

(c) $-2 \cos 2x + \frac{27}{e^{3x}} + C$

(d) $-8 \cos 2x + \frac{3}{e^{3x}} + C$

(e) $2 \cos 2x - \frac{3}{e^{3x}} + C$

6. Newton's method is used to estimate the x -coordinate of the point of intersection of the curves $y = x^3 + 3x$ and $y = 4x + 1$. If we start with $x_0 = 1$, then $x_1 =$

(a) 1.5

(b) 1.05

(c) 1.005

(d) 1.4

(e) 1.75

7. $\frac{d}{dx} [\csc^2(e^x) - \cot^2(e^x)] =$

(a) 0

(b) e^x

(c) $3e^x \cot(e^x) \cdot \csc^2(e^x)$

(d) $-2e^x$

(e) $-e^x \csc(e^x) \cdot \cot(e^x)$

8. If the length x of an edge of a cube decreases at the rate of $0.05 \text{ cm}/\text{min}$, then the surface area of the cube when $x = 5 \text{ cm}$ is decreasing at the rate of

(a) $3 \text{ cm}^2/\text{min}$

(b) $6 \text{ cm}^2/\text{min}$

(c) $0.6 \text{ cm}^2/\text{min}$

(d) $2 \text{ cm}^2/\text{min}$

(e) $0.2 \text{ cm}^2/\text{min}$

9. If $y'' = \frac{3}{\sqrt{x}}$, $y'(4) = 8$ and $y(1) = 0$, then $y(16) =$
- (a) 192
 - (b) 132
 - (c) 64
 - (d) 100
 - (e) 36
10. The sum of two nonnegative numbers ^{is} ~~in~~ 27. If p is the product of one of the numbers and the square root of the other, then the largest possible value of p is
- (a) 54
 - (b) 64
 - (c) 48
 - (d) 44
 - (e) 36

11. The function $f(x) = 5x^3 - 3x^5$ has
- (a) 2 local extrema and 3 points of inflection
 - (b) 2 local extrema and 2 points of inflection
 - (c) 2 critical points and 3 points of inflection
 - (d) 3 critical points and no points of inflection
 - (e) 2 local extrema and one point of inflection
12. If $f'(x) = \frac{x^2(x-1)}{x+2}$, then which one of the following statement is **TRUE**?
- (a) f has one local minimum and no local maxima
 - (b) f has one local minimum and one local maximum
 - (c) f is increasing on $(-2, 1)$
 - (d) f is decreasing on $(1, \infty)$
 - (e) f has one local maximum and no local minima

13. If c is a number which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$ on the interval $[0, 2]$, then $\pi^2 c^2 + 16 =$

(a) $4\pi^2$

(b) 0

(c) π^2

(d) $2\pi^2$

(e) 4

14. Let f be a differentiable function such that $1 < f'(x) < 3$ for all values of x , then which one of the following statement is TRUE ?

(a) $2 < f(7) - f(5) < 6$

(b) $3 < f(7) - f(5) < 6$

(c) $2 < f(7) - f(5) < 3$

(d) $1 < f(7) - f(5) < 3$

(e) $5 < f(7) - f(5) < 7$

15. The sum of the absolute maximum and the absolute minimum values of the function

$$f(x) = 2 \cos x + 2 \cos^2 x, \quad \frac{\pi}{2} \leq x \leq 2\pi$$

is

- (a) $\frac{7}{2}$
 - (b) $\frac{5}{2}$
 - (c) 0
 - (d) -1
 - (e) 1
16. Given that $f(x) = \frac{x^2 + 1}{\sqrt{2x + 1}}$, then the number of critical points of f is

- (a) 1
- (b) 2
- (c) 3
- (d) 0
- (e) 4

17. The linearization of $f(x) = x^x$ at $x = 1$ is given by

(a) $L(x) = x$

(b) $L(x) = x + 1$

(c) $L(x) = 2x - 1$

(d) $L(x) = -x + 2$

(e) $L(x) = x^x$

18. If $f(x) = \log_2(8x^{\ln 2})$ then $f'\left(\frac{1}{2}\right) =$

(a) 2

(b) $\ln 2$

(c) $-\ln 2$

(d) -2

(e) 8

19. Evaluate $\lim_{x \rightarrow -\infty} \frac{-3x + 1}{\sqrt{4x^2 + 6x + 1}}$.

(a) $\frac{3}{2}$

(b) $\frac{-3}{2}$

(c) ∞

(d) $-\infty$

(e) $\frac{-3}{4}$

20. The equation of the tangent line to the curve $y^{\sin x} = \tan x$ at the point $\left(\frac{\pi}{4}, 1\right)$, is

(a) $y - 1 = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(b) $y - 1 = \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$

(c) $y - 1 = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

(d) $y - 1 = \frac{1}{2\sqrt{2}}\left(x - \frac{\pi}{4}\right)$

(e) $y - 1 = -\frac{1}{2\sqrt{2}}\left(x - \frac{\pi}{4}\right)$

21. If $f(x) = \frac{x + |x|}{|x|}$, then which one of the following statements is **TRUE** ?

(a) $\lim_{x \rightarrow 0^-} f'(x) = 0$

(b) $\lim_{x \rightarrow \infty} f'(x) = 1$

(c) $\lim_{x \rightarrow -\infty} f'(x) = 1$

(d) $\lim_{x \rightarrow 0^+} f'(x) = \infty$

(e) $\lim_{x \rightarrow 0} f'(x) = \pm 1$

22. Which one of the following functions has a graph with a vertical tangent at $x = 0$?

(a) $f(x) = x - x^{1/3}$

(b) $f(x) = x - x^{-1/3}$

(c) $f(x) = \frac{1}{|x|}$

(d) $f(x) = x - x^{4/3}$

(e) $f(x) = \frac{x^2}{x^4 - x^2}$

23. If $x^4 + y^4 = 16$, then $y'' =$

(a) $\frac{-48x^2}{y^7}$

(b) $\frac{3x^3}{y^2}$

(c) $4x^3 + 4$

(d) $12(x^2 + y^2)$

(e) $\frac{y^3}{4x^3}$

24. Which one of the following statements is **FALSE** about the graph of the function

$$f(x) = -x - \frac{1}{x-1}?$$

- (a) The graph of f has exactly one inflection point
- (b) The graph of f is concave downward on $(1, \infty)$
- (c) The graph of f is concave upward on $(-\infty, 1)$
- (d) The line $y = -x$ is an oblique asymptote of the graph of f
- (e) The line $x = 1$ is a vertical asymptote to the graph of f

25. The height of a right circular cone of largest volume that can be inscribed in a sphere of radius 1 is

[Hint: The volume of a cone = $\frac{1}{3} \times (\text{area of base}) \times \text{height}$]

(a) $\frac{4}{3}$

(b) $\frac{3}{2}$

(c) $\frac{5}{3}$

(d) $\frac{5}{4}$

(e) $\frac{7}{4}$

26. $\sum_{k=3}^{100} (2 - 4k + k^2)$ is equivalent to

(a) $\sum_{n=1}^{98} (-2 + n^2)$

(b) $\sum_{n=1}^{102} (-2 + 2n + n^2)$

(c) $\sum_{n=1}^{98} (-4 + n^2)$

(d) $\sum_{n=1}^{102} (-2 + 4n + n^2)$

(e) $\sum_{n=1}^{98} (-6 + n^2)$

27. Which one of the following statements is **TRUE** about the continuity of the function?

$$f(x) = \begin{cases} \frac{x^3 - 1}{x^2 + 3x - 4}, & x \neq 1, -4 \\ 4, & x = 1 \\ 5, & x = -4 \end{cases}$$

- (a) f has a removable discontinuity at $x = 1$
- (b) f has a jump discontinuity at $x = 1$
- (c) f has an infinite discontinuity at $x = 1$
- (d) f is continuous at $x = -4$
- (e) f has a removable discontinuity at $x = -4$

28. Evaluate $\lim_{x \rightarrow \infty} (1 + 4x)^{3/\ln x}$.

- (a) e^3
- (b) e^{12}
- (c) $e^{3/4}$
- (d) 1
- (e) Does not exist.