King Fahd University of Petroleum and Minerals
Math & Stat. Departement
Quiz (2)

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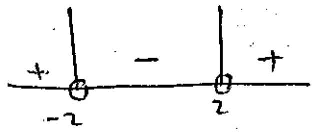
Q1) Let $f(x)=\dfrac{-x+2}{\sqrt{x^2-4}}$. Use the concept of limits to find all vertical and horizontal asymptotes (if any).



V.A : $x^2-4=0 \Rightarrow x=\pm 2$

D_f : $(-\infty,-2)\cup(2,\infty)$.

Since $\lim\limits_{x\to -2^-}\dfrac{-x+2}{\sqrt{x^2-4}}=\infty$ $\therefore$ $x=-2$ is V.A

but $\lim\limits_{x\to 2^+}\dfrac{-x+2}{\sqrt{(x-2)(x+2)}}=\lim\limits_{x\to 2^+}\dfrac{-\cancel{(x-2)}\sqrt{x-2}}{\cancel{(x-2)}\sqrt{x+2}}=0$

$\Rightarrow$ $x=2$ is Not vertical Asymptote.

H.A : $\lim\limits_{x\to\infty}\dfrac{-x+2}{\sqrt{x^2-4}}=\lim\limits_{x\to\infty}\dfrac{-1+\frac{2}{x}}{\sqrt{1-\frac{4}{x^2}}}=-1$

$\lim\limits_{x\to-\infty}\dfrac{-x+2}{\sqrt{x^2-4}}=\lim\limits_{x\to-\infty}\dfrac{-1+2/x}{-\sqrt{1-\frac{4}{x^2}}}=1$

$\therefore$ $y=1$ and $y=-1$ are H.A.

Q2) If $|f(x)+3|<|x-5|$, then find $\lim\limits_{x\to 5}f(x)$. (Hint: Use the Squeezing theorem)

$-|x-5|<f(x)+3<|x-5| \Rightarrow -3-|x-5|<f(x)<-3+|x-5|$

Since $\lim\limits_{x\to 5}(-3-|x-5|)=-3=\lim\limits_{x\to 5}(-3+|x-5|)$

$\therefore$ by sequeezing Theorem $\lim\limits_{x\to 5}f(x)=-3$.

Q3) where is $f(x)=\dfrac{\ln(2-x)}{\sqrt{2+x}}$ continuous?

$\ln(2-x)$ is cont. over its domain $(-\infty,2)$.

$\sqrt{2+x}$ " " " " " $[-2,\infty)$

$\Rightarrow$ f is cont. on $(-\infty,2)\cap[-2,\infty)-\{-2\}$

$=(-2,2)$.

Q4) Let $f(x)=\begin{cases}\sqrt{x+2} & \text{if } 0\le x\le 2\\ x^3-2x & \text{if } x>2\end{cases}$. Is f continuous at $x=2$?. If not, what kind of dicontiuity does it have at $x=2$. Justify your answer.

$f(2)=2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \sqrt{x+2} = 2$

and $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^3-2x = 4$.

Since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x) \Rightarrow \lim_{x\to 2} f(x)$ DNE

and f has Jump discontinuity.

Q5) Use the Intermediate Value Theorem to show that the graphs of the functions $g(x)=3x^3-5x+7$ and $h(x)=x^3-4x+9$ intersects over the interval $[1,2]$.

let $f(x) = g(x) - h(x) = 2x^3 - x - 2$ Continuous on $[1,2]$ Poly.

$f(1) = -1 < 0$, $f(2) = 12 > 0$, $f(1) \neq f(2)$.

Since $f(1) < N=0 < f(2) \Rightarrow$ by I.V.T there is a $c \in (1,2)$ such that $f(c)=0 \Leftrightarrow g(c)-h(c)=0$

$\Leftrightarrow g(c)=h(c)$ so, they intersect at c.

Q6) Using the ε,δ definition of limits, prove that $\lim_{x\to 1}\left(-1+\frac{3}{2}x\right)=\frac{1}{2}$

Step I: let $\epsilon>0$ be given. We need to find $\delta>0$ s.t

$\left|-1+\frac{3}{2}x-\frac{1}{2}\right|<\epsilon$ whenever $0<|x-1|<\delta$.

$\left|\frac{3}{2}x-\frac{3}{2}\right|<\epsilon$ " "

$\frac{3}{2}|x-1|<\epsilon$ " "

$|x-1|<\frac{2}{3}\epsilon$ " $\therefore$ Take $\delta=\frac{2}{3}\epsilon$

step II: If $|x-1|<\frac{2}{3}\epsilon$ then $\left|-1+\frac{3}{2}x-\frac{1}{2}\right|$

$= \left|\frac{3}{2}x-\frac{3}{2}\right| = \frac{3}{2}|x-1| < \frac{3}{2}\cdot\frac{2}{3}\epsilon$

$=\epsilon$