

1. Using the definition of derivative, evaluate

$$\lim_{x \rightarrow 3} \frac{(33-2x)^{-2/3} - (1/9)}{x-3}$$

i. Function: $f(x) = (33-2x)^{-2/3}$

$$a = 3$$

$$f(a) = (33-6)^{-2/3}$$

$$= (27)^{-2/3} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

ii) $\lim_{x \rightarrow 3} \frac{(33-2x)^{-2/3} - 1/9}{x-3}$

$$= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x-a}$$

$$= f'(a) = f'(3)$$

iii) $f'(x) = \frac{-2}{3}(33-2x)^{-5/3}(-2)$

iv) $f'(3) = \frac{-4}{3}(27)^{-5/3}$

$$= \frac{-4}{3}\left(\frac{1}{3}\right)^5 = \frac{-4}{729}$$

Ans $\lim_{x \rightarrow 3} \frac{(33-2x)^{-2/3} - (1/9)}{x-3}$

$$= \frac{-4}{729}$$

2. Find an equation of the normal line to the graph

of the curve $G(u) = \frac{(u+4)(u-2)}{u^3+2}$ when $u=1$.

1) $G(u) = \frac{u^2 + 2u - 8}{u^3 + 2}$

2) $G'(u) = \frac{(u^3+2)(2u+2) - (u^2+2u-8)3u^2}{(u^3+2)^2}$

3) $G'(1) = \frac{(3)(4) - (-5)3}{(3)^2} = 3$

4) Slope of normal line when $u=1$:
 $= -1/3$

(5) Eq. of normal line:

$$y - y_0 = m(u - u_0)$$

$$u_0 = 1$$

$$y_0 = G(u_0) = G(1)$$

$$= \frac{-5}{3}$$

$$\boxed{y + \frac{5}{3} = -\frac{1}{3}(u-1)}$$

1. Using the definition of derivative, evaluate

$$\lim_{x \rightarrow -2} \frac{(7x+6)^{-1/3} + (1/2)}{x+2}$$

i- Function: $f(x) = (7x+6)^{-1/3}$
 $a = -2$
 $f(a) = (-14+6)^{-1/3}$
 $= (-8)^{-1/3} = -1/2$

(ii) $\lim_{x \rightarrow -2} \frac{(7x+6)^{-1/3} + (1/2)}{x+2}$

$$= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$= f'(a) = f'(-2)$$

(iii) $f'(x) = \frac{-1}{3} (7x+6)^{-4/3} \cdot 7$

(iv) $f'(-2) = \frac{-7}{3} (-14+6)^{-4/3}$
 $= \frac{-7}{3} (-8)^{-4/3}$
 $= \frac{-7}{3} \left(\frac{1}{16}\right)$
 $= \frac{-7}{48}$

Ans

$$\lim_{x \rightarrow -2} \frac{(7x+6)^{-1/3} + (1/2)}{x+2} = \frac{-7}{48}$$

2. Find an equation of the normal line to the graph

of the curve $K(w) = \frac{(w+1)(w-2)}{w^2+1}$ when $w=1$.

1) $K(w) = \frac{w^2 - w - 2}{w^2 + 1}$

2) $K'(w) = \frac{(w^2+1)(2w-1) - (w^2-w-2)2w}{(w^2+1)^2}$

3) $K'(1) = \frac{(2)(1) - (-2)(2)}{(2)^2} = 3/2$

4) Slope of normal line when $w=1$
 $= -2/3$

⑤ Eq. of normal line:

$$y - y_0 = m(w - w_0) \quad w_0 = 1$$

$$y_0 = K(w_0)$$

$$= K(1)$$

$$= \frac{(2)(-1)}{1+1}$$

$$= -1$$

$$y + 1 = \frac{-2}{3}(w - 1)$$