ID#______ Section:_____

Quiz 7 (optional) MATH 101-T121

Serial #_____

1. Let $x_1 = 3$ is the first approximation of $\sqrt{7}$. Applying the Newton's method on suitable function, find the second approximation.

1) Function: $x = \sqrt{7} = 7^{1/2}$

$\Rightarrow \quad x^2 = 7 \Rightarrow x^2 - 7 = 0$

2) Put $F(x) = x^2 - 7$ and approx. it zeros: Note: $F'(x) = 2x$

3) $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 3 - \frac{2}{6}$

$= 3 - \frac{1}{3}$

$= \frac{8}{3}$

2. Solve the initial value problem:

$$\frac{d^2y}{dx^2} = \cos\pi x, \quad \left.\frac{dy}{dx}\right|_{x=1} = 1, \quad y(1) = 2$$

(i) $\int \frac{d^2y}{dx^2}\,dx = \int \cos(\pi x)\,dx$

$\frac{dy}{dx} = \frac{1}{\pi}\sin(\pi x) + C$

$1 = \left.\frac{dy}{dx}\right]_{x=1} = \frac{1}{\pi}\sin(\pi) + C$

$C = 1$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{\pi}\sin(\pi x) + 1}$

(ii) $\int \frac{dy}{dx}\,dx = \int \left(\frac{1}{\pi}\sin(\pi x) + 1\right)dx$

$y = \frac{-1}{\pi 2}\cos(\pi x) + x + C$

$2 = y(1) = \frac{-1}{\pi 2}\cos\pi + 1 + C$

$= \frac{1}{\pi 2} + 1 + C$

$\Rightarrow C = 1 - \frac{1}{\pi^2}$

$\Rightarrow \boxed{y = \frac{-1}{\pi^2}\cos(\pi x) + x + 1 - \frac{1}{\pi 2}}$

ID#_______ Section:_____ Quiz 7 (optional) MATH 101-T121 Serial #_____

1. Let $x_1 = 1$ is the first approximation of the point of intersection of two functions

$$f(x) = x^2 \text{ and } g(x) = x+1.$$

Using the Newton's method, find the third approximation.

1) $f(x) = g(x) \Rightarrow x^2 = x+1 \Rightarrow x^2 - x - 1 = 0$

2) Put $F(x) = x^2 - x - 1$ and approx. its zeros. Note: $F'(x) = 2x - 1$

3) $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 1 + \frac{1}{1} = \boxed{2}$

3) $x_3 = x_2 - \frac{F(x_2)}{F'(x_2)} = 2 - \frac{4-2-1}{4-1}$

$= 2 - \frac{1}{3}$

$= \frac{5}{3}$

2. Find the position function $s(t)$ of a particle moving along a straight line if its velocity is $v(t) = \frac{3}{t\sqrt{t^2-1}}$ when $s(2) = 0$

$\frac{d}{dt} s(t) = v(t)$

$s(t) = \int v(t)\,dt$

$= \int \frac{3\,dt}{t\sqrt{t^2-1}}$

$\boxed{s(t) = 3\sec^{-1} t + C}$

Initial Cond

$0 = s(2) = 3\sec^{-1}(2) + C$

$= 3\left(\frac{\pi}{3}\right) + C$

$\Rightarrow C = -\pi$

Ans $s(t) = 3\sec^{-1}(t) - \pi$

Note $\sec^{-1} t = y$

$t = \sec y$

$2 = \sec y$

$2 = \frac{1}{\cos y}$

$\Rightarrow \cos y = \frac{1}{2}$

$y = \frac{\pi}{3}$