Math 101 Quiz 2 Date 29/09/2012

Ex1: Find $\lim\limits_{x \to 1^-} \dfrac{4-6x^3}{(x-2)(x-1)} = ?$

Solution: $\lim\limits_{x \to 1^-} 4-6x^3 = 4-6 = -2$. On the other hand: $\lim\limits_{x \to 1^-} (x-1) = 0^-$ and $\lim\limits_{x \to 1^-} (x-2) = -1 < 0$. So

$$\lim_{x \to 1^-} (x-2)(x-1) = 0^+. \text{ Therefore}$$

$$\lim_{x \to 1^-} \frac{4-6x^3}{(x-2)(x-1)} = -\infty.$$

Ex2: Find all the asymptotes of $f(x) = \dfrac{x^3-x+1}{x^2-4}$.

Solution: • It is easy to see that

$\lim\limits_{x \to \pm 2} f(x) = \pm\infty$. Thus $x=2$ and $x=-2$ are two vertical asymptotes for f

• $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{x^3}{x^2} = \lim\limits_{x \to +\infty} x = +\infty$; also $\lim\limits_{x \to -\infty} f(x) = -\infty$.

Hence f has no vertical asymptotes.

• The long division of the numerator by the denominator implies:

$$x^3 - x + 1 = x(x^2-4) + 3x + 1. \text{ Thus:}$$

$$f(x) = \frac{x^3-x+1}{x^2-4} = x + \frac{3x+1}{x^2-4}. \text{ Now:}$$

$$\lim_{x \to \pm\infty} \left[f(x) - x \right] = \lim_{x \to \pm\infty} \frac{3x+1}{x^2-4} = \lim_{x \to \pm\infty} \frac{3x}{x^2} = \lim_{x \to \pm\infty} \frac{3}{x} = 0$$

Hence the line $y = x$ is an oblique asymptote for $f(x)$ at $\pm\infty$.

Ex3: Find $\lim_{x \to -\infty} \sqrt{x^2+1} + x - 2 = ?$

Solution:

$$\lim_{x \to -\infty} \sqrt{x^2+1} + x - 2 = \lim_{x \to -\infty} \left[\frac{(\sqrt{x^2+1} + x)(\sqrt{x^2+1} - x)}{\sqrt{x^2+1} - x} \right] - 2$$

$$= \lim_{x \to -\infty} \left[\frac{x^2+1-x^2}{\sqrt{x^2+1} - x} - 2 \right] = \lim_{x \to -\infty} \left(\frac{1}{\sqrt{x^2+1} - x} - 2 \right)$$

Since $\lim_{x \to -\infty} \sqrt{x^2+1} - x = +\infty$, we have that

$$\lim_{x \to -\infty} \frac{1}{\sqrt{x^2+1} - x} = 0.$$ Therefore:

$$\lim_{x \to -\infty} \left(\frac{1}{\sqrt{x^2+1} - x} - 2 \right) = -2$$