Math 101 Quiz 6 Date 01/12/2012

Ex1: Let $f(x) = \tan^{-1}(e^{-2x})$.

a) Find the linearization of $f(x)$ at 0

b) Use part a) to approximate $f(\frac{\pi}{12})$

Solution:

a) By the Chain rule, $f'(x) = \dfrac{-2e^{-2x}}{1+(e^{-2x})^2} = -\dfrac{2e^{-2x}}{1+e^{-4x}}$.

Thus $f'(0) = \dfrac{-2}{2} = -1$. Hence the linearization of f at 0 is

$L(x) = f(0) + f'(0)(x-0) = \tan^{-1}(1) - x = \dfrac{\pi}{4} - x$.

b) $f(x) \approx L(x)$ when x is in a neighborhood of 0. Thus

$f(\frac{\pi}{12}) \approx \dfrac{\pi}{4} - \dfrac{\pi}{12} = \dfrac{\pi}{6}$.

Ex2: Find the absolute extrema of $g(x) = -\sqrt{4-x^2}$ over the closed interval $[-2,2]$.

Solution: g is continuous on $[-2,2]$. For every $x \in (-2,2)$ we have: $g'(x) = \dfrac{x}{\sqrt{4-x^2}}$. Thus $g'(x) = 0$ iff $x = 0$. Hence g has only one critical point $x = 0$ inside $(-2,2)$. Now $g(-2) = g(2) = 0$ and $g(0) = -2$. Therefore 0 is the absolute min. and both -2 and 2 are the absolute max.