Serial No.:______ Student Name: Key Student Number: ______

Instructor: M. Z. Abu-Sbeih Math 201.11 – Q3 Date: 7-11-2012

Show all your work. No credits for answers not supported by work.
Write neatly and eligibly. You may loose credits for messy work.

Problem 1: (20 Points)

10 (a) Find the volume of the parallelepiped determined by the vectors $\vec{A} = \langle 3,-5,1\rangle, \vec{B} = \langle 2,-2,0\rangle, \vec{C} = \langle 3,1,1\rangle$.

$$V = |\vec{A}\cdot\vec{B}\times\vec{C}| = \begin{vmatrix} 3 & -5 & 1 \\ 2 & -2 & 0 \\ 3 & 1 & 1 \end{vmatrix} = 12$$

10 (b) Find the equation of the plane tangent to the sphere $x^2 + y^2 + z^2 - 2x + 2y - 2z = 0$ at the point P(2,0,2).

The standard form of the equation: $(x-1)^2 + (y+1)^2 + (z-1)^2 = 3$

Center is $C(1,-1,1)$.

Note that the vector $\vec{PC} = \langle 1-2, -1-0, 1-2\rangle = \langle -1,-1,-1\rangle$ is perpendicular to the tangent plane at the point P.

Hence the equation of the plane is

$$-(x-2) - (y-0) - (z-2) = 0$$

OR $x + y + z = 4$

Problem 2: (20 Points) Consider the line $L: \frac{x-1}{2} = \frac{y-2}{3} = z - 1$ and the plane $P: x - y + z = 6$.

7 (a) Show that the line is parallel to the plane.

A vector in the direction of the line is $\vec{u} = \langle 2,3,1\rangle$

" " normal to the plane is $\vec{v} = \langle 1,-1,1\rangle$.

Now: $\vec{u}\cdot\vec{v} = \langle 2,3,1\rangle \cdot \langle 1,-1,1\rangle = 2-3+1 = 0$.

$\therefore$ This means that the line L is perpendicular to the normal vector to the plane.

Thus the Line is parallel to the plane.

Note: Another way is to show that the line does not intersect the plane.

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(b) Does the plane contain the line? Show why.

No, because the point $(1,2,1)$ is on the line L but not in the plane:

$$1-2+1 \stackrel{?}{\neq} 6$$

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(c) If the plane does not contain the line, what is the distance between them.

The distance between them is equal to the distance from any point on the line, say $P(1,2,1)$, and the plane. Using the formula:

$$d = \frac{|1-2+1-6|}{\sqrt{1^2+(-1)^2+1^2}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

Problem 3: (20 Points) Consider the surface presented by the equation $4x^2 - 3y^2 + 12z^2 + 12 = 0$.

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(a) Find the traces with the three coordinate planes. Identify each curve.

equation in standard form: $\frac{y^2}{4} - \frac{x^2}{3} - z^2 = 1$

Trace in the xy-plane (put $z=0$): $\frac{y^2}{4} - \frac{x^2}{3} = 1$ hyperbola

" " " yz-plane (put $x=0$): $\frac{y^2}{4} - z^2 = 1$ hyperbola

" " " xz-plane (put $y=0$):

$$-\frac{x^2}{3} - z^2 = 1$$

No point satisfies this equation, so NO Trace.

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(b) Identify the surface.

The surface is hyperboloid of two sheets

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(c) Sketch the surface.