Show all your work. No credits for answers not supported by work. Write neatly and eligibly. You may lose credits for messy work.

**Problem 1: (20 Points)**

(a) Find the volume of the parallelepiped determined by the vectors \( \vec{A} = (3,-5,1) \), \( \vec{B} = (2,-2,0) \), \( \vec{C} = (3,1,1) \).

\[
\sqrt{\vec{A} \cdot \vec{B} \times \vec{C}} = \begin{vmatrix} 3 & -5 & 1 \\ 2 & -2 & 0 \\ 3 & 1 & 1 \end{vmatrix} = 12
\]

(b) Find the equation of the plane tangent to the sphere \( x^2 + y^2 + z^2 - 2x + 2y - 2z = 0 \) at the point \( P(2,0,2) \).

The standard form of the equation: \((x-1)^2 + (y+1)^2 + (z-1)^2 = 3\)

Center is \( C(1,-1,1) \).

Note that the vector \( \overrightarrow{PC} = (-1-2,-1-0,1-2) = (-3,-1, -1) \) is perpendicular to the tangent plane at the point \( P \).

Hence the equation of the plane is

\[-(x-2) - (y-0) - (z-2) = 0\]

OR \[x + y + z = 4\]

**Problem 2: (20 Points)** Consider the line \( L: \frac{x-1}{2} = \frac{y-2}{3} = z - 1 \) and the plane \( P: x - y + z = 6 \).

(a) Show that the line is parallel to the plane.

A vector in the direction of the line is \( \vec{U} = \langle 2, 3, 1 \rangle \)

normal to the plane is \( \vec{V} = \langle 1, -1, 1 \rangle \).

Now: \( \vec{U} \cdot \vec{V} = \langle 2, 3, 1 \rangle \cdot \langle 1, -1, 1 \rangle = 2 - 3 + 1 = 0 \).

\( \therefore \) This means that the line \( L \) is perpendicular to the normal vector to the plane.

Thus the line is parallel to the plane.

Note: Another way is to show that the line does not intersect the plane.
(b) Does the plane contain the line? Show why.

No, because the point (1, 2, 1) is on the line but not in the plane:

\[ 1 - 2 + 1 = 0 \neq 6 \]

(c) If the plane does not contain the line, what is the distance between them.

The distance between them is equal to the distance from any point on the line, say \( P(1, 2, 1) \), and the plane.

Using the formula:

\[
 d = \frac{|1 - 2 + 1 - 6|}{\sqrt{1^2 + (-1)^2 + 1^2}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}
\]

**Problem 3: (20 Points)** Consider the surface presented by the equation \( 4x^2 - 3y^2 + 12z^2 + 12 = 0 \).

(a) Find the traces with the three coordinate planes. Identify each curve.

- **Equation in standard form:** \( \frac{y^2}{4} - \frac{x^2}{3} - z^2 = 1 \)

  - **Trace in the xy-plane (put z=0):** \( \frac{y^2}{4} - \frac{x^2}{3} = 1 \) hyperbola
  - **Trace in the yz-plane (put x=0):** \( \frac{y^2}{4} - z^2 = 1 \) hyperbola
  - **Trace in the xz-plane (put y=0):** \( -\frac{x^2}{3} - z^2 = 1 \)

  No point satisfies this equation, so NO Trace.

(b) Identify the surface.

The surface is a **hyperboloid of two sheets**.

(c) Sketch the surface.