**King Fahd University of Petroleum & Minerals**
**Department of Mathematics & Statistics**
**Math 301 Major Exam 1**
**The First Semester of 2011-2012 (121)**

**Time Allowed: 120 Minutes**

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Name: ____________________ ID#: ____________

Instructor: ____________________ Sec #: ________ Serial #: ________

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- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

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| Question # | Marks | Maximum Marks |
|---|---|---|
| 1 | | 14 |
| 2 | | 12 |
| 3 | | 14 |
| 4 | | 16 |
| 5 | | 12 |
| 6 | | 16 |
| 7 | | 16 |
| Total | | 100 |

**Q:1** (7+7 points) Given the position vector $\vec{r}(t) = 2\sqrt{2}\, t\, \hat{\imath} + e^{2t}\hat{\jmath} + e^{-2t}\hat{k}$ of a curve $C$:

**(a)** Find a unit tangent vector to the curve at $t = 0$.

**(b)** Find the length of the curve for $0 \le t \le 1$.

a) $r'(t)$ is a tangent vector, so that, $\vec{u} = \frac{r'(0)}{\|r'(0)\|}$ (2pts)

is a unit tangent vector to the curve at $t=0$.

$r'(t) = 2\sqrt{2}\,\vec{i} + 2e^{2t}\vec{j} - 2e^{-2t}\vec{k} \longrightarrow$ (1pt)

$r'(0) = 2\sqrt{2}\,\vec{i} + 2\vec{j} - 2\vec{k} \longrightarrow$ (1pt)

$\|r'(0)\| = 2\sqrt{(\sqrt{2})^2 + 1 + 1} = 4 \longrightarrow$ (1pt)

$\boxed{\vec{u} = \frac{\sqrt{2}}{2}\vec{i} + \frac{1}{2}\vec{j} - \frac{1}{2}\vec{k}} \longrightarrow$ (2pts)

b)

$s = \int_0^1 \|r'(t)\|\, dt \longrightarrow$ (2pts)

$= 2\int_0^1 \sqrt{2 + e^{4t} + e^{-4t}}\, dt \longrightarrow$ (1pt)

$= 2\int_0^1 \sqrt{(e^{2t} + e^{-2t})^2}\, dt \longrightarrow$ (1pt)

$= 2\int_0^1 (e^{2t} + e^{-2t})\, dt$

$= \left[e^{2t} - e^{-2t}\right]_0^1$

$\boxed{s = e^2 - e^{-2}}$

(3pts)

**Q:2** (6+6 points) Let $f(x,y,z) = xy^2 + 2x^2y + z^2$.

**(a)** Find the directional derivative of $f(x,y,z)$ at $(-1,1,2)$ in the direction of $\hat{\imath} - \hat{\jmath} + 3\hat{k}$.

**(b)** Find the direction and value of maximum directional derivative of $f(x,y,z)$ at $(1,0,1)$.

a) $\nabla f(x,y,z) = (y^2 + 4xy)\vec{i} + (2xy + 2x^2)\vec{j} + 2z\,\vec{k}$ 3pts

$\nabla f(-1,1,2) = -3\,\vec{i} + 4\,\vec{k}$ → 1pt

The unit vector in the direction of $\vec{i} - \vec{j} + 3\vec{k}$

is $\vec{u} = \frac{1}{\sqrt{1+1+9}}(\vec{i} - \vec{j} + 3\vec{k})$

$= \frac{1}{\sqrt{11}}\vec{i} - \frac{1}{\sqrt{11}}\vec{j} + \frac{3}{\sqrt{11}}\vec{k}$ → 1pt

The directional derivative of $f(x,y,z)$ at $(-1,1,2)$ in the direction of $\vec{i} - \vec{j} + 3\vec{k}$ is

$D_{\vec{u}} f(-1,1,2) = \nabla f(-1,1,2) \cdot \vec{u}$

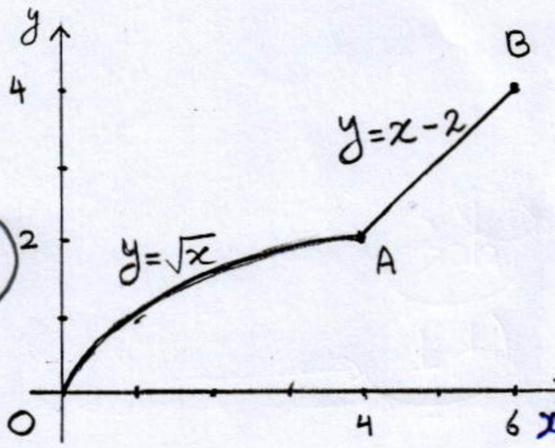
$= \frac{-3}{\sqrt{11}} + \frac{12}{\sqrt{11}}$

$\boxed{D_{\vec{u}} f(-1,1,2) = \frac{9}{11}\sqrt{11}}$ 1pt

b.) The direction of maximum directional derivative is $\nabla f(1,0,1) = 2\vec{j} + 2\vec{k}$. 3pts

The maximum directional derivative at $(1,0,1)$ is $\|\nabla f(1,0,1)\| = 2\sqrt{2}$. 3pts

**Q:3 (a)** (8 points) Evaluate the line integral $\int_C (x-3y)dx + ydy$ along the path from O to B as shown in the figure:




a) $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, where

$\mathcal{C}_1 : 0 \le x \le 4,\ y = \sqrt{x}$

$\mathcal{C}_2 : 4 \le x \le 6,\ y = x-2$

1pt

$$\int_{\mathcal{C}_1} (x-3y)\,dx + y\,dy = \int_0^4 (x - 3\sqrt{x})\,dx + \sqrt{x}\,\frac{dx}{2\sqrt{x}}$$

3pts

$$= \int_0^4 \left(x - 3\sqrt{x} + \frac{1}{2}\right) dx = \left[\frac{x^2}{2} - 2x^{3/2} + \frac{x}{2}\right]_0^4 = -6$$

$$\int_{\mathcal{C}_2} (x-3y)\,dx + y\,dy = \int_4^6 [x - 3(x-2)]\,dx + (x-2)\,dx$$

3pts

$$= \int_4^6 (-x+4)\,dx = \left[-\frac{x^2}{2} + 4x\right]_4^6 = -2$$

$$\Rightarrow \boxed{\int_{\mathcal{C}} (x-3y)\,dx + y\,dy = -8}$$

1pt

**(b)** (6 points) Evaluate the line integral $\int_C x^2\,dx + (x+z)\,dy + y^2\,dz$, where the path $C$ is given by $x = t,\ y = t^2,\ z = -t,\ 0 \le t \le 1$.

$$\int_{\mathcal{C}} x^2 dx + (x+z)\,dy + y^2 dz = \int_0^1 t^2 dt + (t-t)\,2dt + t^4(-dt)$$

3pts

$$= \int_0^1 (t^2 - t^4)\,dt$$

$$= \left[\frac{t^3}{3} - \frac{t^5}{5}\right]_0^1 = \frac{1}{3} - \frac{1}{5}$$

$$= \frac{2}{15}$$

3pts