King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 301 Major Exam 2
The First Semester of 2012-2013 (121)

Time Allowed: 120 Minutes

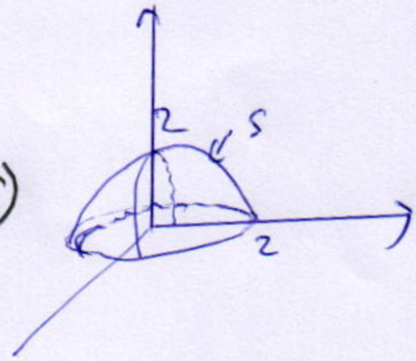
Name: ______________________________ ID#: ________________

Instructor: ________________________ Sec #: ________ Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		16
3		16
4		12
5		14
6		16
7		14
Total		100

Q:1 (12 points) Let $\overrightarrow{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ be a vector field and S the surface of the region bounded by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the plane $z = 0$. Use divergence theorem to evaluate $\int\int_S (\overrightarrow{F} \cdot \hat{n}) dS$.



The Divergence theorem says

$$\iint_S F\cdot n\,dS = \iiint_D \operatorname{div} F\, dV \quad (3\,pts)$$

$$\operatorname{div} F = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial y}(y^3) + \frac{\partial}{\partial z}(z^3)$$
$$= 3x^2 + 3y^2 + 3z^2 \quad (3\,pts)$$

$$\iint_S F\cdot n\,dS = 3\iiint_D (x^2+y^2+z^2)\, dV$$

We may use spherical coordinates. This gives

$$\iint_S F\cdot n\,dS = 3\int_0^{2\pi}\int_0^{\pi/2}\int_0^2 \rho^2\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta \quad (2\,pts)$$

$$\left.\begin{aligned} &= 3\int_0^{2\pi}\int_0^{\pi/2} \sin\phi \left[\frac{\rho^5}{5}\right]_0^2 d\phi\, d\theta \\ &= 3\cdot\frac{32}{5}\int_0^{2\pi}\left[-\cos\phi\right]_0^{\pi/2} d\theta \end{aligned}\right\} (2\,pts)$$

$$= \frac{192\pi}{5} \quad (2\,pts)$$

Q:2 (4+6+6 points) Find the following:

(a) $\mathcal{L}\{e^{-t}\cosh t\}$,

(b) $\mathcal{L}^{-1}\left\{\frac{s-1}{s^2+2s}\right\}$,

(c) $\mathcal{L}^{-1}\left\{\frac{s+1}{(s-1)(s^2+1)}\right\}$.

a) $\mathcal{L}\{e^{-t}\cosh t\} = \frac{s+1}{(s+1)^2-1} = \frac{s+1}{s(s+2)}$ 4pts

b) $\frac{s-1}{s^2+2s} = \frac{s-1}{s(s+2)} = \frac{-1/2}{s} + \frac{\frac{3}{2}}{s+2}$ 3pts

$\mathcal{L}^{-1}\left\{\frac{s-1}{s^2+2s}\right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\}$

$= -\frac{1}{2} + \frac{3}{2}e^{-2t}$ 3pts

c) $\frac{s+1}{(s-1)(s^2+1)} = \frac{1}{s-1} - \frac{s}{s^2+1}$ 3pts

$\mathcal{L}^{-1}\left\{\frac{s+1}{(s-1)(s^2+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$

$= e^{t} - \cos t$ 3pts

Q:3 (4+5+7 points) Find the following:

(a) $\mathcal{L}\{te^{-t}\cos t\}$,

(b) $\mathcal{L}\{f(t)\}$, where $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2, & t \ge 1 \end{cases}$.

(c) $\mathcal{L}^{-1}\left\{\dfrac{s^2}{(s^2+1)^2}\right\}$.

a) $\mathcal{L}\{te^{-t}\cos t\} = -\dfrac{d}{ds}\mathcal{L}\{e^{-t}\cos t\} = -\dfrac{d}{ds}\left[\dfrac{s+1}{(s+1)^2+1}\right] = \dfrac{s^2+2s}{(s^2+2s+2)^2}$ 2pts 2pts 4pts

b) $f(t) = t - t\,\mathcal{U}(t-1) + 2\,\mathcal{U}(t-1)$ 2pts

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t\} - \mathcal{L}\{t\,\mathcal{U}(t-1)\} + \mathcal{L}\{2\,\mathcal{U}(t-1)\}$

$= \dfrac{1}{s^2} - e^{-s}\mathcal{L}\{t+1\} + e^{-s}\mathcal{L}\{2\}$

$= \dfrac{1}{s^2} - \left(\dfrac{1}{s^2} + \dfrac{1}{s}\right)e^{-s} + \dfrac{2}{s}e^{-s}$

$= \dfrac{1}{s^2} - \left(\dfrac{1}{s^2} - \dfrac{1}{s}\right)e^{-s}$ 3pts

c) $\dfrac{s^2}{(s^2+1)^2} = \dfrac{s}{s^2+1} \cdot \dfrac{s}{s^2+1}$

$\mathcal{L}^{-1}\left\{\dfrac{s^2}{(s^2+1)^2}\right\} = \cos t * \cos t$ 2pts

$= \int_0^t \cos\tau \cos(t-\tau)\,d\tau$ 1pt

$= \dfrac{1}{2}\int_0^t [\cos t + \cos(2\tau - t)]\,d\tau$

$= \dfrac{1}{2}\left(t\cos t + \left[\dfrac{\sin(2\tau - t)}{2}\right]_0^t\right)$

$= \dfrac{1}{2}\left(t\cos t + \dfrac{\sin t}{2} + \dfrac{\sin t}{2}\right)$ 2pts

$= \dfrac{1}{2}(t\cos t + \sin t)$ 2pts