MATH 301.2 (Term 121)

Quiz 1 (Sects. 9.8, 9.9) Duration: 20mn

Name: **ID number:**

1.)(5pts)Evaluate $\int_C (x+y)dx + ydy$, **where** C **is the triangle with vertices** $(-1.0), (-1,1), (1,1)$, **oriented in the counterclockwise direction.**

2.)(5pts)Show that $F(x,y) = (xy+\cos y)\vec{\imath} + (\frac{1}{2}x^2 - x\sin y)\vec{\jmath}$ **is a conservative field and find its potential. Then, evaluate** $\int_{(0,0)}^{(1,2)} F.dr$.

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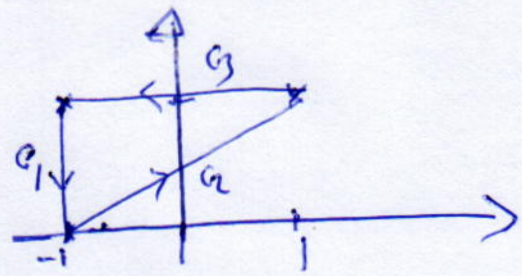
1)



C_1: $x=-1$, $0 \le y \le 1$

C_2: $-1 \le x \le 1$, $y = \frac{1}{2}x + \frac{1}{2}$

C_3: $-1 \le x \le 1$, $y = 1$,

$$\int_C (x+y)\,dx + y\,dy = \int_1^0 y\,dy + \int_{-1}^{1} \left(x + \tfrac{1}{2}x + \tfrac{1}{2}\right) dx + \int_{-1}^{1} \left(\tfrac{1}{2}x + \tfrac{1}{2}\right)\tfrac{1}{2}\,dx + \int_1^{-1} (x+1)\,dx$$

$$= \left[\frac{y^2}{2}\right]_1^0 + \left[\frac{3}{8}x^2 - \frac{x}{4}\right]_{-1}^{1}$$

$$= -\frac{1}{2} - \frac{1}{2}$$

$$= -1$$

2) $P(x,y) = xy + \cos y$

$Q(x,y) = \frac{1}{2}x^2 - x\sin y$

$\frac{\partial Q}{\partial x} = x - \sin y$, $\frac{\partial P}{\partial y} = x - \sin y$

$\Rightarrow F$ is conservative.

$F = \nabla \phi$

$$\begin{cases} \frac{\partial \phi}{\partial x} = xy + \cos y & (1) \\ \frac{\partial \phi}{\partial y} = \frac{1}{2}x^2 - x\sin y & (2) \end{cases}$$

$(1) \Rightarrow \phi(x,y) = \frac{x^2}{2}y + x\cos y + f(y)$

$(2) \Rightarrow \frac{x^2}{2} - x\sin y + f'(y) = \frac{1}{2}x^2 - x\sin y$

$f'(y) = 0$, $f(y) = C$

$$\boxed{\phi(x,y) = \frac{x^2}{2}y + x\cos y + C}$$

$$\int_{(0,0)}^{(1,2)} F.dr = \phi(1,2) - \phi(0,0)$$

$$= 1 + \cos 2$$

MATH 301.3 (Term 121)

Quiz 1 (Sects. 9.8, 9.9) Duration: 20mn

Name: ID number:

1.)(5pts)Evaluate $\int_C (x+y)dx + ydy$, where $\mathcal{C}$ is the curve given by $x = 2\cos t$ and $y = 2\sin t$ for $0 \le t \le \pi$.

2.)(5pts)Show that $\int_C (x^2 y+1)\, dx + (\frac{1}{3}x^3 + y^2)dy$ is independent of path. Then, evaluate the integral along any path from $(0,0)$ to $(1,2)$.

1) $\int_C (x+y)\,dx + y\,dy = \int_0^{\pi} 2(\cos t + \sin t)(-2\sin t)\,dt + 2\sin t\,(2\cos t)\,dt$

$= -4\int_0^{\pi} \sin^2 t\, dt$

$= -4\int_0^{\pi} \frac{1-\cos 2t}{2}\, dt$

$= -2\left[t - \frac{\sin 2t}{2}\right]_0^{\pi}$

$= -2\pi$

2) $P(x,y) = x^2 y+1$

$Q(x,y) = \frac{1}{3}x^3 + y^2$

$\frac{\partial Q}{\partial x} = x^2, \quad \frac{\partial P}{\partial y} = x^2.$

$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow F = P\vec{i} + Q\vec{j}$ is conservative.

Thus, $F = \nabla\phi$

$\begin{cases} \frac{\partial \phi}{\partial x} = x^2 y + 1 & (1) \\ \frac{\partial \phi}{\partial y} = \frac{1}{3}x^3 + y^2 & (2) \end{cases}$

$(1) \Rightarrow \phi(x,y) = \frac{x^3}{3}y + x + f(y)$

$(2) \Rightarrow \frac{x^3}{3} + f'(y) = \frac{1}{3}x^3 + y^2$

$f'(y) = y^2, \quad f(y) = \frac{y^3}{3} + C$

$\boxed{\phi(x,y) = \frac{x^3}{3}y + x + \frac{y^3}{3} + C}$

$\int_{(0,0)}^{(1,2)} (x^2 y+1)\,dx + y\,dy = \phi(1,2) - \phi(0,0)$

$= \frac{2}{3} + 1 + \frac{8}{3}$

$= \frac{13}{3}$