MATH 301.2 (Term 121)

Quiz 3 (Sects. 4.1-4.3) Duration: 20mn

Name: **ID number:**

1.)(4pts)Solve the IVP $y'' - 2y' + 5y = e^t$, $y(0) = 0$, $y'(0) = 1$

2.)(3pts)Find $L^{-1}\{\frac{s+1}{s(s-1)(s+2)}\}$.

3.)(3pts)Find $L\{f(t)\}$, **where** $f(t) = \begin{cases} 1, & 0 \le t < 2, \\ t^2, & t \ge 2. \end{cases}$

Let $F(s) = \mathcal{L}\{y\}$.

1) $\mathcal{L}\{y'' - 2y' + 5y\} = \mathcal{L}\{e^t\}$

$s^2F(s) - sy(0) - y'(0) - 2[sF(s) - y(0)] + 5F(s) = \frac{1}{s-1}$

$(s^2 - 2s + 5)F(s) = 1 + \frac{1}{s-1} = \frac{s}{s-1}$

$F(s) = \frac{s}{(s-1)(s^2-2s+5)}$

$= \frac{a}{s-1} + \frac{bs+c}{s^2-2s+5}$

We set $a = \frac{1}{4}$, $b = -\frac{1}{4}$, $c = \frac{5}{4}$.

Thus, $F(s) = \frac{\frac{1}{4}}{s-1} + \frac{-\frac{1}{4}s + \frac{5}{4}}{(s-1)^2 + 4}$

$= \frac{\frac{1}{4}}{s-1} + \frac{-\frac{1}{4}(s-1)}{(s-1)^2+4} + \frac{1}{(s-1)^2+4}$

$\Rightarrow \boxed{y(t) = \frac{1}{4}e^t - \frac{1}{4}e^t\cos 2t + \frac{1}{2}e^t\sin 2t}$

2.) $\frac{s+1}{s(s-1)(s+2)} = \frac{a}{s} + \frac{b}{s-1} + \frac{c}{s+2}$

We find $a = -\frac{1}{2}$, $b = \frac{2}{3}$, $c = -\frac{1}{6}$

$\frac{s+1}{s(s-1)(s+2)} = \frac{-\frac{1}{2}}{s} + \frac{\frac{2}{3}}{s-1} + \frac{-\frac{1}{6}}{s+2}$

$\Rightarrow \boxed{\mathcal{L}^{-1}\left\{\frac{s+1}{s(s-1)(s+2)}\right\} = -\frac{1}{2} + \frac{2}{3}e^t - \frac{1}{6}e^{-2t}}$

3.) $f(t) = 1 - u(t-2) + t^2u(t-2)$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{1\} - \mathcal{L}\{u(t-2)\} + \mathcal{L}\{t^2u(t-2)\}$

$= \frac{1}{s} - e^{-2s}\mathcal{L}\{1\} + e^{-2s}\mathcal{L}\{(t+2)^2\}$

$= \frac{1}{s} - \frac{e^{-2s}}{s} + e^{-2s}\mathcal{L}\{t^2 + 4t + 4\}$

$= \frac{1}{s} - \frac{e^{-2s}}{s} + e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$

$\Rightarrow \boxed{\mathcal{L}\{f(t)\} = \frac{1}{s} + \left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{3}{s}\right)e^{-2s}}$

MATH 301.3 (Term 121)

Quiz 3 (Sects. 4.1-4.3) Duration: 20mn

Name: **ID number:**

1.)(4pts)Solve the IVP $y'' + 2y' + 3y = e^{-t}$, $y(0) = 0$, $y'(0) = 1$

2.)(3pts)Find $L^{-1}\{\frac{s-1}{s(s+1)(s+2)}\}$.

3.)(3pts)Find $L\{f(t)\}$, **where** $f(t) = \begin{cases} 0, & 0 \le t < 1, \\ t^2, & 1 \le t < 2, \\ 0, & t \ge 2. \end{cases}$

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1) let $F(s) = \mathcal{L}\{y\}$.

$\mathcal{L}\{y'' + 2y' + 3y\} = \mathcal{L}\{e^{-t}\}$

$s^2 F(s) - s y(0) - y'(0) + 2[sF(s) - y(0)] + 3F(s) = \frac{1}{s+1}$

$(s^2 + 2s + 3) F(s) = 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$

$F(s) = \frac{s+2}{(s+1)(s^2+2s+3)}$

$= \frac{a}{s+1} + \frac{bs+c}{s^2+2s+3}$

We find $a = \frac{1}{2}$, $b = -\frac{1}{2}$, $c = 1/2$

$F(s) = \frac{\frac{1}{2}}{s+1} + \frac{-\frac{1}{2}s + \frac{1}{2}}{(s+1)^2 + 2}$

$= \frac{\frac{1}{2}}{s+1} + \frac{-\frac{1}{2}(s+1)}{(s+1)^2+2} + \frac{1}{(s+1)^2+2}$

$\Rightarrow \boxed{y = \frac{1}{2}e^{-t} - \frac{1}{2}e^{-t}\cos\sqrt{2}t + \frac{1}{\sqrt{2}}e^{-t}\sin\sqrt{2}t}$

2.) $\frac{s-1}{s(s+1)(s+2)} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{s+2}$

We find $a = -\frac{1}{2}$, $b = 2$, $c = -\frac{3}{2}$

$\frac{s-1}{s(s+1)(s+2)} = \frac{-\frac{1}{2}}{s} + \frac{2}{s+1} - \frac{3/2}{s+2}$

$\Rightarrow \boxed{L^{-1}\left\{\frac{s-1}{s(s+1)(s+2)}\right\} = -\frac{1}{2} + 2e^{-t} - \frac{3}{2}e^{-2t}}$

3) $f(t) = t^2[u(t-1) - u(t-2)]$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 u(t-1)\} - \mathcal{L}\{t^2 u(t-2)\}$

$= e^{-s}\mathcal{L}\{(t+1)^2\} - e^{-2s}\mathcal{L}\{(t+2)^2\}$

$= e^{-s}\mathcal{L}\{t^2+2t+1\} - e^{-2s}\mathcal{L}\{t^2+4t+4\}$

$\boxed{= \left(\frac{2}{s^3} + \frac{2}{s} + \frac{1}{s}\right)e^{-s} - \left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)e^{-2s}}$