1. (5pts) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem $y'' + \lambda y = 0$, $y(0) + y'(0) = 0$, $y(1) = 0$.

2. (5pts) Expand $f(x) = x$, $0 < x < 1$, in a Fourier Bessel series using Bessel functions of order one satisfying the boundary conditions $J_1(\alpha) + \frac{\alpha}{\beta} J'_1(\alpha) = 0$.

3. (5pts) Write the first four terms of the Fourier-Legendre expansion of $f(x) = \begin{cases} x, & -1 \leq x < 0, \\ 0, & 0 < x \leq 1. \end{cases}$
3) \[ f(x) = \frac{1}{\pi} \sum_{n=0}^{\infty} c_n P_n(x), \quad (n = \frac{2n+1}{2}) \]
\[ c_n = \frac{2}{L} \int_{-L}^{L} f(x) P_n(x) \, dx \]

\[ c_0 = \frac{1}{2} \int_{-\pi}^{\pi} f(x) \, dx = \frac{1}{L} \int_{-L}^{L} x \, dx = -\frac{1}{4L} \]

\[ c_1 = \frac{2}{L} \int_{-\pi}^{\pi} f(x) \, dx = \frac{2}{L} \int_{-\pi}^{\pi} x^2 \, dx = \frac{1}{2} \]

\[ c_2 = \frac{5}{2} \int_{-\pi}^{\pi} f(x) \, dx = \frac{5}{2} \int_{-\pi}^{\pi} (3x^4 - x) \, dx = 0 \]

\[ c_3 = \frac{7}{2} \int_{-\pi}^{\pi} f(x) \, dx = \frac{7}{2} \int_{-\pi}^{\pi} (5x^6 - 3x^4) \, dx = 0 \]
1. (5pts) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem
\[ y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(1) = 0. \]
2. (5pts) Expand \( f(x) = x, \quad 0 < x < 3 \), in a Fourier Bessel series using Bessel functions of order one satisfying the boundary conditions \( J_1(3\alpha) + a J'_1(3\alpha) = 0 \)
3. (5pts) Write the first four terms of the Fourier-Legendre expansion of
\[ f(x) = \begin{cases} 
0, & -1 \leq x < 0, \\
-x, & 0 < x \leq 1.
\end{cases} \]