Exercise 1

Solve the following general system by inverting the coefficient matrix

\begin{align*}
x_1 + 3x_2 + x_3 &= 4 \\
2x_1 + 2x_2 + x_3 &= -1 \\
2x_1 + 3x_2 + x_3 &= 3
\end{align*}
Exercise 2

Are there values of $r$ and $s$ for which
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & r - 2 & 2 \\
0 & s - 1 & r + 2 \\
0 & 0 & 3
\end{bmatrix}
\]
has rank 1 or 2? If so, find those values.
Exercise 3

Let

\[
A = \begin{bmatrix}
5 & 2 & -2 \\
2 & 5 & -2 \\
-2 & -2 & 5
\end{bmatrix}.
\]

(a) Verify that \( \det(\lambda I_3 - A) \), the characteristic polynomial of \( A \), is given by

\[
(\lambda - 3)^2(\lambda - 9).
\]

(b) Find a non-singular matrix \( P \) such that \( P^{-1}AP = \text{diag}(3, 3, 9) \).