

Math 440 : Final Exam -121

- 1) a) Give the definition of a smooth surface in \mathbb{R}^3
b) Show that the surface $x^2 + y^2 - z^2 = 1$ is a smooth surface , and compute its transition functions
- 2) Prove that the curvature of a plane curve $y = f(x)$ is given by $\kappa = \frac{f''}{(1+f'^2)^{\frac{3}{2}}}$
- 3) Using the parametrization
 $r(\theta, z) = (\cos\theta, \sin\theta, z)$ for a cylinder of radius 1, write down the geodesic equations and solve them.
- 4) a) Define the second fundamental form and compute it for the surface of revolution obtained by rotating the curve $y = h(u), z = u$ in the (y, z) plane about $z - axis$
b) Find the principal curvatures for the surface in part (b)
- 5) Show that the 2-sphere minus a point can be mapped 1: 1 and onto the plane in a differentiable Way.