1. Let $X = C([0,1]; \mathbb{R})$ and let $K : X \to X$ be the integral operator defined by

$$Ku(t) = \int_0^t (t-s)u(s)ds.$$ 

Show that $K$ is compact and describe its spectrum.

2. Consider the integral operator $K$ with kernel $k(t,s) = \frac{1}{|t-s|^{\alpha}}$ and acting on $C([a,b]; \mathbb{R})$.

For what values of $\alpha$ is $K$ bounded and compact?

3. Find the eigenvalues and eigenfunctions of the integral equation

$$u(t) = \lambda \int_0^t \frac{e^{-(t-s)}}{t} u(s)ds, \ t \geq 0.$$ 

Are the eigenvalues isolated? Are the eigenfunctions bounded for $t \geq 0$?

4. Solve the integral equation

$$u(t) = \lambda \int_0^1 (1 + ts) u(s)ds + f(t).$$ 

5. Find the eigenvalues of

$$\phi(t) = \lambda \int_{-\pi}^{\pi} \sum_{n=0}^{\infty} k_n \cos nt \cos(n+1)s \phi(s) ds,$$

assuming that the eigenfunctions are of the form $\sum_{n=0}^{\infty} c_n \cos nt$. 