1. Consider the integral equation
\[ u(x) = \lambda \int_0^1 \left( xt - \frac{x^2}{2} \right) u(t) dt + f(x), \quad 0 \leq x \leq 1. \] (1)
(i) Find the eigenvalues and eigenfunctions of the homogeneous equation.
(ii) When \( \lambda \) is an eigenvalue, take \( f(x) = ax + b \). What values of \( a \) and \( b \) are permissible in order that (1) is solvable.
(iii) Find the general solution of (1).

2. Consider the integral equation
\[ u(x) = f(x) + \lambda \int_0^1 e^{x-t} u(t) dt. \] (2)
(i) Compute the iterated kernels and find the resolvent kernel \( R(x, t; \lambda) \).
(ii) Find the eigenvalues and eigenfunctions.
(iii) What is the general solution of (2)

3. Let \( X = C([-\pi, \pi]; \mathbb{R}) \) be the Banach space of continuous functions equipped with the sup-norm. Define an integral operator \( K : X \to X \) by
\[ Ku(x) = \int_{-\pi}^{\pi} \sin(x + t) u(t) dt. \]
(i) Show that \( K \) is compact and symmetric.
(ii) Describe the spectrum of \( K \)
(iii) What are the eigenfunctions of \( K \).
(iv) Solve the integral equation \( u = \lambda Ku + \sigma, \sigma \in [-\pi, \pi] \).

4. Show that the integral equation
\[ u(x) = \lambda \int_0^\pi \sin x \cos u(t) dt \]
has no eigenvalues.