Quiz 1 AS381

1) The random loss variable X has a pdf given by
$f(x) = 1/50 \qquad 0 < x < 50.$

a) Calculate $E[X]$ and $Var[X]$

b) Consider a proportional policy where $I(x) = kx \quad 0 < k < 1$
and a stop-loss policy where
$I(x) = \{ \begin{matrix} 0 & x < d \\ x - d & x \geq d. \end{matrix}$
Determine k and d such as the pure premium in each case is P=6.25.

a) Solution for a.
$E[X] = \int_0^{50} xf(x)dx = \int_0^{50} \frac{x}{50}dx = 25.$
$E[X^2] = \int_0^{50} x^2 f(x)dx = \int_0^{50} \frac{x^2}{50}dx = \frac{2500}{3}$
$Var[X] = E[X^2] - (E[X])^2 = \frac{2500}{3} - 25^2 = \frac{625}{3} = 208.33$

b) Solution for b
Pure premium means no security loading, so $\beta = P = 6.25$
For proportional policy, $E[I(X)] = \beta = 6.25 = \int_0^{50} kxf(x)dx = \int_0^{50} \frac{kx}{50}dx = 25k = 6.25 \rightarrow k = 0.25$
For stop-loss policy, $E[I(X)] = \beta = 6.25 = \int_d^{50}(x - d)f(x)dx = \int_d^{50}(1 - F(x))dx$ implies
$\int_d^{50}(1 - \frac{x}{50})dx = \frac{1}{100}d^2 - d + 25 = \frac{1}{100}(d - 50)^2 = 6.25 \rightarrow d = 25.$

2) A portfolio contains 36 independent policies with benefit amount $B = 1$ each. For each policy the probability q of a claim is 1/6. Let S be the total claims for the portfolio. Using a normal approximation, estimate Pr(S > 9).

Solution: $E[S] = \sum_{i=1}^{36} E[X_i] = nBq = 36B(1/6) = 6$
$Var[S] = \sum_{i=1}^{36} Var[X_i] = nB^2q(1 - q) = 36B^2(1/6)(1 - 1/6) = 5$
$Pr(S > 6) = Pr(\frac{S-E[S]}{\sqrt{Var(s)}} > \frac{9-6}{\sqrt{5}}) = Pr(Z > 1.3416) = 1 - .9099 = 0.090\,1$

3) Let X_i for $i = 1, 2$ be independent and identically distributed with the pdf
$f(x) = \{ \begin{matrix} 1 & 0 \leq x < 1 \\ 0 & elsewhere. \end{matrix}$
Let $S = X_1 + X_2$.

a) Find $F_S(s)$

b) Find $Pr(S < 1.5)$

a) Solution for a

Important regions are $0 \leq S < 1$, and $1 \leq S < 2$
From equation (2.3.6),$F_S(s) = \int_0^s f_x(s - y)f_y(y)dy$

$$F(x) = \left\{ \begin{matrix} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geqslant 1 \end{matrix} \right| \qquad f(y) = \{ \begin{matrix} 1 & 0 \leq y < 1 \\ 0 & elsewhere \end{matrix}$$

$$F_s(s) = \left\{ \begin{matrix} 0 & s < 0 \\ \int_0^s F(x)f(y)dy = \int_0^s x(1)dy = \int_0^s (s-y)(1)dy = \frac{1}{2}s^2 & 0 \leq s < 1 \\ \int_0^{s-1} F(x)f(y)dy + \int_{s-1}^1 F(x)f(y)dy = \int_0^{s-1} 1(1)dy + \int_{s-1}^1 (s-y)(1)dy = 2s - \frac{1}{2}s^2 - 1 & 1 \leq s < 2 \\ 1 & s \geq 2 \end{matrix} \right|$$

$$F_s(s) = \left\{ \begin{matrix} 0 & s < 0 \\ \frac{1}{2}s^2 & 0 \leq s < 1 \\ 2s - \frac{1}{2}s^2 - 1 = \left(-\frac{1}{2}\right)\left(s^2 - 4s + 2\right) & 1 \leq s < 2 \\ 1 & s \geq 2 \end{matrix} \right| :$$

$$f_S(s) = F_s^{'}(s) = \left\{ \begin{array}{cc} 0 & s < 0 \\ \frac{d}{ds}\frac{1}{2}s^2 = s & 0 \le s < 1 \\ \frac{d}{ds}\left(-\frac{1}{2}\left(s^2 - 4s + 2\right)\right) = 2 - s & 1 \le s < 2 \\ \frac{d}{ds}1 = 0 & s \ge 2 \end{array} \right|$$

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b) Solution for b

$Pr(S < 1.5) = \left(-\frac{1}{2}\right)\left(1.5^2 - 4(1.5) + 2\right) = 0.875$

Further note

If we extend this to $S = X_1 + X_2 + X_3$, important regions are $0 \le S < 1$, $1 \le S < 2$ and $2 \le S < 3$ and with

$$F^{(2)}(x) = \left\{ \begin{array}{cc} 0 & s < 0 \\ \frac{1}{2}s^2 & 0 \le s < 1 \\ \left(-\frac{1}{2}\right)\left(s^2 - 4s + 2\right) & 1 \le s < 2 \\ 1 & s \ge 2 \end{array} \right| : : \qquad f(y) = \left\{ \begin{array}{cc} 0 & elsewhere \\ 1 & 0 \le y < 1 \end{array} \right|$$

So, from equation (2.3.6), $F_S(s) = \int_0^s F_x(s-y) f_y(y) dy$ we should get the results in exercise 2.8