Solutions

KFUPM Term (122) Name____________________ Serial#__________

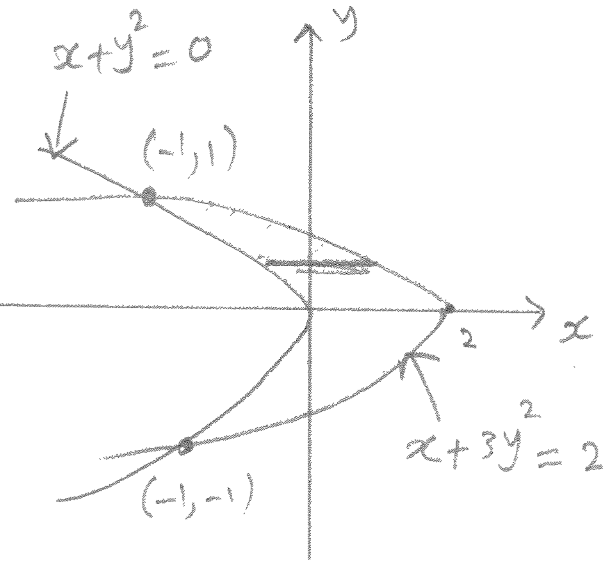
MATH 102 Quiz # 2 ID#________________ Section_____

1) (7-points) Find the integral which represents the area of the region bounded by the curves $x + y^2 = 0$ and $x + 3y^2 = 2$

Points of intersection:

$-y^2 + 3y^2 = 2 \Rightarrow y^2 = 1$

$\Rightarrow \boxed{y = 1, x = -1}, \boxed{y = -1, x = -1}$



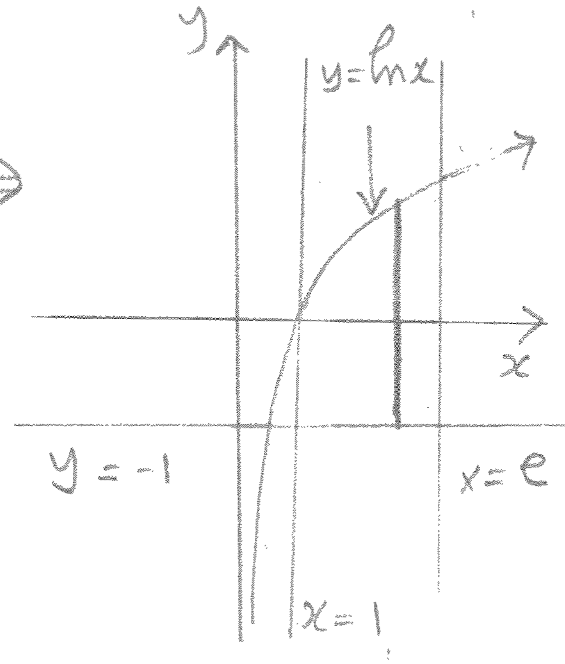
$$\text{Area} = \int_{-1}^{1} [(2 - 3y^2) - (-y^2)]\, dy$$

$$= \int_{-1}^{1} (2 - 2y^2)\, dy$$

2) (6-points) The base of a solid is the region bounded by the curve $y = \ln x$ and the lines $x = 1, x = e$, and $y = -1$. If cross-sections by planes perpendicular to the $x-$ axis are squares, find the integral which represents the volume of the solid.

length of side of a square

$= (\ln x) - (-1) = \ln x + 1 \Rightarrow$



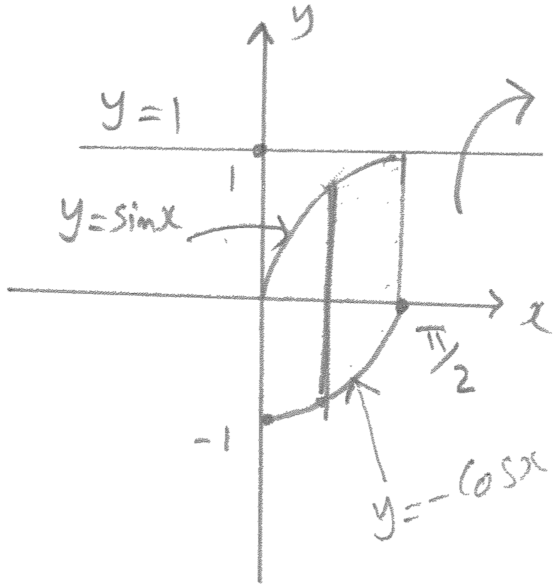
The required Volume

$$= \int_1^e (\ln x + 1)^2\, dx$$

$$= \int_1^e [(\ln x)^2 + 2(\ln x) + 1]\, dx$$

3) (7-points) The region bounded by the curves $y = \sin x, y = -\cos x$ and the lines $x = 0$ and $x = \frac{\pi}{2}$ is revolved about the line $y = 1$. Find the integral which represents the volume of the solid generated.

We use the washer method



outer radius $= 1 - (-\cos x)$

$= 1 + \cos x$

Inner radius $= 1 - \sin x$

The required volume

$$= \int_0^{\pi/2} \pi \left[(1+\cos x)^2 - (1-\sin x)^2 \right] dx$$

$$= \int_0^{\pi/2} \pi \left[\cos^2 x - \sin^2 x + 2\cos x + 2\sin x \right] dx.$$

Solutions

KFUPM Term (122) Name____________________ Serial#__________

MATH 102 Quiz # ID#________________ Section_____

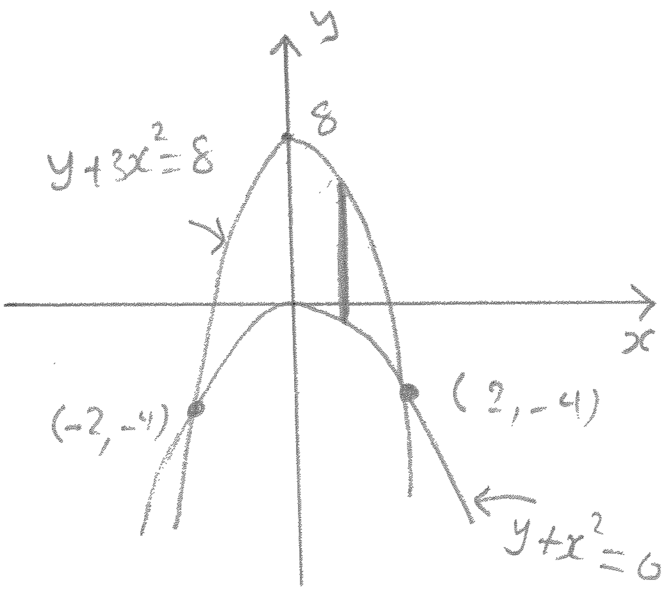
Version (2)

1) (7-points) Find the integral which represents the area of the region bounded by the curves $y + x^2 = 0$ and $y + 3x^2 = 8$.

Points of intersection:

$$-x^2 + 3x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow$$

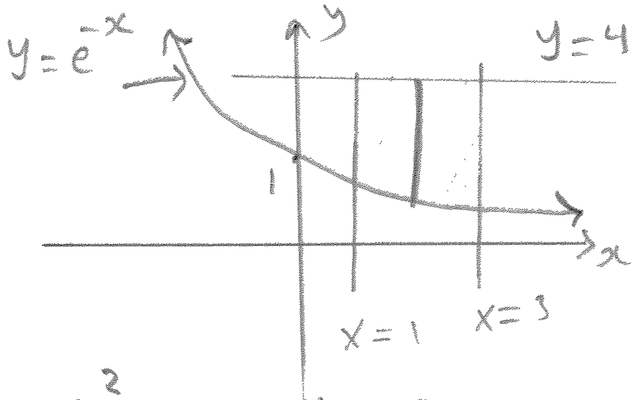
$\boxed{x = -2, y = -4}$ $\boxed{x = 2, y = -4}$



$$\text{Area} = \int_{-2}^{2} \left[(8 - 3x^2) - (-x^2) \right] dx$$

$$= \int_{-2}^{2} (8 - 2x^2)\, dx$$

2) (6-points) The base of a solid in the region bounded by the curve $y = e^{-x}$ and the lines $x = 1, x = 3$, and $y = 4$. If cross-sections by planes perpendicular to the $x-$axis are squares, find the integrals which represents the volume of the solid.



Legth of a sided of a square

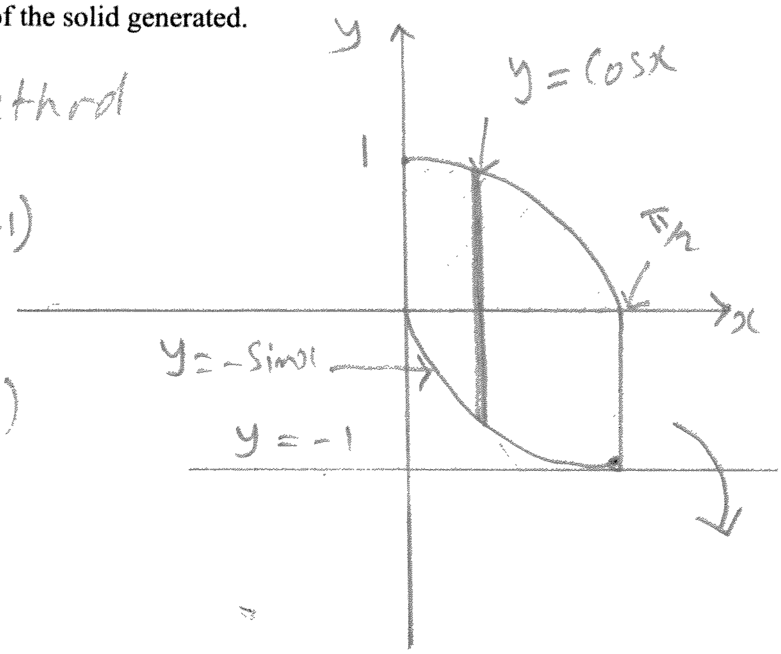
$= 4 - e^{-x} \Longrightarrow$

The required Volume

$$= \int_{1}^{2} (4 - e^{-x})^2\, dx = \int_{1}^{2} (16 - 8e^{-x} + e^{-2x})\, dx$$

3) (2-points) The region bounded by the curves $y = \cos x$, $y = -\sin x$ and the lines $x = 0$ and $x = \frac{\pi}{2}$ is revolved about the line $y = -1$. Find the integral which represents the volume of the solid generated.

We use the Washer Method



outer radius $= \cos x - (-1)$

$= \cos x + 1$

inner radius $= \sin x - (-1)$

$= \sin x + 1$

The required volume

$$= \int_0^{\pi/2} \pi \left[(\cos x + 1)^2 - (\sin x + 1)^2 \right] dx.$$

$$= \int_0^{\pi/2} \pi \left(\cos^2 x - \sin^2 x + 2\cos x - 2\sin x \right) dx.$$