KFUPM Term (122) Name Solutions Serial#_________

MATH 102-01 Quiz # 4 ID#________________ Section_____

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1) (5-points each) Test each of the following improper integrals for convergence or divergence. If converges then find its value.

A) $\int_0^5 \frac{1}{(x-1)^{4/3}}\,dx$

This is an improper integral since its integrand is discontinuous at $x = 1 \in [0,5]$

We first check $\lim_{t\to 1^-}\int_0^t (x-1)^{-4/3}\,dx = \lim_{t\to 1^-}\left[-3(x-1)^{-1/3}\right]_0^t$

$= \lim_{t\to 1^-}\left[\frac{-3}{(t-1)^{-1/3}} + 3\right] = -\infty \Rightarrow \int_0^1 \frac{1}{(x-1)^{4/3}}\,dx$ diverges

$\Rightarrow$ the given improper integral diverges.

B) $\int_0^2 \frac{3x}{\sqrt{4-x^2}}\,dx$

This is an improper integral since its integrand is discontinuous at $x = 2 \in [0,2]$.

We check $K = \lim_{t\to 2^-}\int_0^t (3x)(4-x^2)^{-1/2}\,dx$

To evaluate $I = \int (3x)(4-x^2)^{-1/2}\,dx$ let

$u = 9 - x^2 \Rightarrow du = -2x\,dx \Rightarrow I = -\frac{3}{2}\int u^{-1/2}\,du \Rightarrow$

$I = -\frac{3}{2}\,\frac{u^{1/2}}{1/2} = -3\sqrt{4-x^2} \Rightarrow K = \lim_{t\to 2^-}\left[-3\sqrt{4-x^2}\right]_0^t$

$\Rightarrow K = \lim_{t\to 2^-}\left[-3\sqrt{4-t^2} + 6\right] = 6 \Rightarrow$ the given integral converges and has the value 6.

2) (5-points each) Test each of the following sequences for convergence or divergence. If converges, then find its limit.

A) $\{\ln(5n) - \ln(3n+7)\}$

$$\lim_{n\to\infty} \{\ln(5n) - \ln(3n+7)\} = \lim_{n\to\infty} \ln\left(\frac{5n}{3n+7}\right)$$

$$= \lim_{n\to\infty} \ln\left(\frac{5}{3+7/n}\right) = \ln\left(\frac{5}{3}\right)$$

$\Longrightarrow$ the sequence converges to $\ln\left(\frac{5}{3}\right)$

C) $\left\{\left(1-\frac{3}{n}\right)^{4n}\right\}$ (Remember that 1^{∞} is an indetermined form)

$\lim_{n\to\infty} \left(1-\frac{3}{n}\right)^{4n}$ is of type 1^{∞}

Let $y = \left(1-\frac{3}{x}\right)^{4x} \Rightarrow \ln y = 4x \ln\left(1-\frac{3}{x}\right)$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(1-3/x)}{1/4x} \overset{H.R}{=} \lim_{x\to\infty} \frac{\frac{3/x^2}{1-3/x}}{-1/4x^2}$$

$$= \lim_{x\to\infty} \frac{-12}{1-3/x} = -12 \Rightarrow \lim_{x\to\infty} y = e^{-12}$$

$$\Rightarrow \lim_{n\to\infty} \left(1-\frac{3}{n}\right)^{4n} = e^{-12}.$$

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MATH 102-12 Quiz # 3 ID#________________ Section_____

1) (5-points each) Test each of the following improper integrals for convergence or divergence. If converges then find its value.

A) $\int_{-4}^{-1} \frac{1}{(x+2)^{7/5}} dx$

This is an improper integral since its integrand is discontinuous at $-2 \in [-4,-1]$.

We first check $\lim_{t \to -2^-} \int_{-4}^{t} (x+2)^{-7/5} dx = \lim_{t\to -2} \left[-\frac{5}{2}(x+2)^{-2/5} \right]_{-4}^{t}$

$= \lim_{t \to -2} -\frac{2}{5} \left[\frac{1}{(t+2)^{2/5}} - \frac{1}{6^{2/5}} \right] = -\infty \Rightarrow \int_{-4}^{-2} (x+2)^{-7/2} dx$

diverges $\Rightarrow$ the improper integral $\int_{-4}^{-1} \frac{1}{(x+2)^{7/5}} dx$ diverges.

B) $\int_0^3 \frac{5x}{\sqrt{9-x^2}} dx$.

This is an improper integral since its integrand is discontinuous at $3 \in [0,3]$

We check $\lim_{t \to 3^-} \int_0^t (5x)(9-x^2)^{-1/2} dx = K$

To evaluate $I = \int 5x(9-x^2)^{-1/2} dx$, let $u = 9-x^2 \Rightarrow$

$du = -2x\,dx \Rightarrow I = -\frac{5}{2} \int u^{-1/2} du = -\frac{5}{2} \frac{u^{1/2}}{1/2} \Rightarrow$

$I = -5\sqrt{9-x^2} \Rightarrow K = \lim_{t \to 3^-} \left[-5\sqrt{9-x^2} \right]_0^t \Rightarrow$

$K = \lim_{t \to 3^-} \left[-5\sqrt{9-t^2} + 15 \right] = 15 \Rightarrow$ the given integral converges and has the value 15.

- 2) (5-points each) Test each of the following sequences for convergence or divergence. If converges, then find its limit.

  A) $\{\ln(4n) - \ln(5n-6)\}$

$$\lim_{n\to\infty}\left[\ln(4n) - \ln(5n-6)\right] = \lim_{n\to\infty}\left[\ln\left(\frac{4n}{5n-6}\right)\right]$$

$$= \lim_{n\to\infty} \ln\left(\frac{4}{5-6/n}\right) = \ln(4/5)$$

$\Rightarrow$ the given sequence converges to $\ln\left(\frac{4}{5}\right)$.

C) $\left\{\left(1-\frac{5}{n}\right)^{3n}\right\}$ (Remember that 1^∞ is an indetermined form)

$\lim_{n\to\infty}\left(1-5/n\right)^{3n}$ is of type 1^∞.

Let $y = \left(1-\frac{5}{x}\right)^{3x} \Rightarrow \ln y = 3x \ln\left(1-\frac{5}{x}\right)$

$$\Rightarrow \lim_{x\to\infty} \ln y = \lim_{x\to\infty} 3\,\frac{\ln\left(1-\frac{5}{x}\right)}{\frac{1}{x}} \quad (0/0 \text{ type})$$

$$\underline{\underline{H.R}} \lim_{x\to\infty} \frac{3\left(\frac{5}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = \lim_{x\to\infty}(-15) = -15$$

$$\Rightarrow \lim_{x\to\infty} y = e^{-15} \Rightarrow \lim_{n\to\infty}\left(1-\frac{5}{n}\right)^{3n} = e^{-15}$$