Math 102 Quiz 1 06/02/2013

Ex1: Evaluate the following indefinite integrals:

a) $I = \int \sqrt{\frac{x^3-3}{x^{11}}}\, dx$, b) $J = \int \frac{1}{\sqrt{x}\, e^{-\sqrt{x}}} \sec^2\left(e^{\sqrt{x}}+1\right) dx$

Ex2 Evaluate $\int_a^b (2x+1)\,dx$ as the limit of Riemann sum. Use a partition that subdivides $[a,b]$ into n subintervals of equal width. Work with the right endpoint of each subinterval.

Solution:

Ex1: a) $I = \int \sqrt{\frac{1}{x^8} - \frac{3}{x^{11}}}\, dx = \int \sqrt{\frac{1}{x^8}} \sqrt{1-\frac{3}{x^3}}\, dx$

$= \int \frac{1}{x^4} \sqrt{1-\frac{3}{x^3}}\, dx$. Let $t = 1 - \frac{3}{x^3}$. Then

$dt = \frac{9}{x^4} dx$. So $\frac{1}{x^4} dx = \frac{1}{9} dt$. Thus:

$I = \int \frac{1}{9}\sqrt{t}\, dt = \frac{2}{27} t^{3/2} + c = \frac{2}{27}\left(1-\frac{3}{x^3}\right)^{3/2} + C$

b) Let $t = e^{\sqrt{x}} + 1$. Then $dt = \frac{1}{2\sqrt{x}} e^{\sqrt{x}} dx$. So

$\frac{1}{\sqrt{x}\, e^{-\sqrt{x}}} dx = 2dt$. Hence: $J = \int 2\sec^2(t)\, dt$

$= 2\tan t + C = 2\tan\left(e^{\sqrt{x}}+1\right) + C$

Ex2. We consider the following partition of $[a,b]$:

$$P=\Big\{\underset{x_0}{\underset{\downarrow}{a}},\ \underset{x_1}{\underset{\downarrow}{a+\frac{b-a}{n}}},\ \underset{x_2}{\underset{\downarrow}{a+2\frac{b-a}{n}}},\ \dots,\ \underset{x_k}{\underset{\downarrow}{a+k\frac{b-a}{n}}},\ \dots,\ \underset{x_n}{\underset{\downarrow}{b}}\Big\}$$

The Riemann sum is given by:

$$S_P=\sum_{k=1}^{n}\left[2\left(a+k\,\frac{b-a}{n}\right)+1\right]\frac{b-a}{n}$$

$$=2a\frac{b-a}{n}\sum_{k=1}^{n}1+2\left(\frac{b-a}{n}\right)^2\sum_{k=1}^{n}k+\frac{b-a}{n}\sum_{k=1}^{n}1$$

$$=2a(b-a)+2\left(\frac{b-a}{n}\right)^2\frac{n(n+1)}{2}+b-a$$

We take the limit when $n\to+\infty$ ie $\|P\|=\frac{b-a}{n}\to 0$:

$$\lim_{n\to+\infty}S_P=2a(b-a)+(b-a)^2+b-a$$

$$=(b-a)\left[2a+b-a+1\right]=(b-a)(b+a+1)$$

$$\text{“}=(b-a)(b+a)+b-a$$

$$=(b^2-a^2)+(b-a)\text{”}$$