Math 102 Quiz 2 13/02/2013

Ex1: Find the area of the region enclosed by:

$$y = 2\sin x \quad , \quad y = \sin 2x \quad \text{and} \quad 0 \leq x \leq \pi.$$

Ex2: Find the area of the region in the 1st quadrant bounded on the left by the y-axis, below by the line $y = \frac{x}{4}$, above left by the curve $y = 1+\sqrt{x}$, and above right by the curve $y = \frac{2}{\sqrt{x}}$.

Solution:

Ex1: For all $x \in [0,\pi]$, we have:

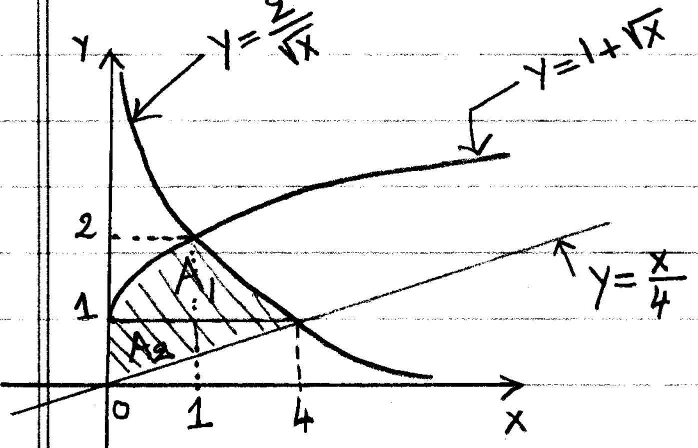
$$2\sin x - \sin 2x = 2\sin x - 2\sin x\cos x = 2\sin x \cdot (1-\cos x)$$

Since $\sin x \geq 0$ for $x \in [0,\pi]$ and $\cos x \leq 1$ for all x, we conclude that $2\sin x \geq \sin 2x$ for all $x \in [0,\pi]$.
Moreover we have equality, i.e., $2\sin x = \sin 2x$ only when $x = 0$ or $x = \pi$. Therefore, the area will be given by:

$$A = \int_0^{\pi} (2\sin x - \sin 2x)\,dx = \left[-2\cos x + \frac{1}{2}\cos 2x\right]_0^{\pi}$$

$$= 2 + \frac{1}{2} + 2 - \frac{1}{2} = 4.$$

Ex2:



*) Clearly: $\frac{2}{\sqrt{x}} = \frac{x}{4}$ implies: $x^{3/2} = 8$ i.e $x = 8^{2/3}$ i.e $x = 4$

*) A_2 is a triangle with area given by: $A_2 = (1\times 4)\frac{1}{2} = 2$

*) To find the area A_1, we will integrate with respect to y. In fact it is easy to see that $y = \frac{2}{\sqrt{x}}$ is on the right side of $y = 1+\sqrt{x}$

First we need to find the y-coordinate of the intersection point: we solve for x, $\frac{2}{\sqrt{x}} = 1 + \sqrt{x}$. Here clearly x must be 1. So the intersection point is $(1, 2)$.

Hence the area A_1 is given by:

$$A_1 = \int_1^2 \left[\frac{4}{y^2} - (y-1)^2\right] dy$$ In fact: "$y = \frac{2}{\sqrt{x}} \Rightarrow x = \frac{4}{y^2}$" and $y = 1 + \sqrt{x} \Rightarrow x = (y-1)^2$.

Therefore: $$A_1 = \left[-\frac{4}{y} - \frac{(y-1)^3}{3}\right]_1^2 = \frac{5}{3}.$$

Conclusion: The total area is:

$$A = A_1 + A_2 = \frac{5}{3} + 2 = \frac{11}{3}$$