Math 102 Quiz 4 04/03/2013

Evaluate the following integrals.

a) $I = \int x^{2x}(1+\ln x)\,dx$

b) $J = \int \frac{1}{x}\cos(1-\log_2 x)\,dx$

c) $K = \int x 2^{x^2}\,dx$

<u>Solution</u>:

a) $I = \int e^{2x\ln x}(1+\ln x)dx$. Let $u = 2x\ln x$. Then

$du = 2\left[\ln x + x.\frac{1}{x}\right]dx = 2(1+\ln x)dx$. Thus

$I = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{2x\ln x} + c$

$= \frac{1}{2}x^{2x} + c$

b) $J = \int \frac{1}{x}\cos\left(1-\frac{\ln x}{\ln 2}\right)dx$. Let $u = \log_2 x = \frac{\ln x}{\ln 2}$. Then

$du = \frac{1}{x\ln 2}dx$. Thus: $J = \int \ln 2.\cos(1-u)\,du$

$= -\ln 2.\sin(1-u) + c = -\ln 2.\sin\left(1-\log_2 x\right) + c$

c) $K = \int x 2^{x^2}dx$. Let $u = x^2$. Then $du = 2x\,dx$. Thus

$K = \frac{1}{2}\int 2^u\,du = \frac{1}{2\ln 2}2^u + c = \frac{1}{2\ln 2}2^{x^2} + c$