1) Given that \( y = c_1 e^t \sin 2t + c_2 e^t \cos 2t \) is a two-parameter family of solutions of the differential equation
\[
y'' - 2y' + 5y = 0
\]
Determine whether a member of the family of the solutions of the above differential equation can be found that satisfies the boundary conditions:
\[
y(0) = 1, \quad y(\pi) = -1
\]

2) Consider the differential equation
\[
y'' - 4y' + 4y = 0
\]
(a) Find the interval in which the two solutions
\[
y_1 = e^{2x} \quad \text{and} \quad y_2 = xe^{2x} \quad \text{are linearly independent.}
\]
(b) Form a general solution of the differential equation
3) Consider the differential equation
\[ y'' + 3y = -18e^{3x} \] (1)
(a) Verify that \( y_1 = \cos \sqrt{3}x \) and \( y_2 = \sin \sqrt{3}x \) are solutions of \( y'' + 3y = 0 \).
(b) Find a particular solution of the differential equation (1) of the form \( y = Ae^{3x} \)
(c) Write the general solution of the differential equation (1).
4) Solve the initial value problem

\[ y''' + y'' - y' - y = 0; \quad y(0) = 0, y'(0) = 0, y''(0) = 4 \]

5) Given that \( y = x^2 + 8x + 33 \) is a particular solution to the differential equation

\[ y'' + 4y' - y = -x^2 + 1 \]

And \( y = -\frac{1}{4}xe^{-x} + \frac{1}{8}e^{-x} \) is a particular solution to the differential equation

\[ y'' + 4y' - y = xe^{-x} - e^{-x} \]

Find a particular solution to the differential equation

\[ y'' + 4y' - y = -x^2 + 1 - 2xe^{-x} + 2e^{-x} \]
6) Find the general solution of the differential equation

\[ xy'' - y' + 4x^3y = 0 \]

Given that \( y_1 = \sin(x^2) \) is a solution