Exercise 1 (10 points 3–3–4):
(1) Find all elements of order 2 of the multiplicative group \( \mathbb{C}^* \).
(2) Find all elements of order 2 of the group \( \mathbb{R}^* \oplus \mathbb{R}^* \).
(3) Use (1) and (2) to show that \( \mathbb{C}^* \) is not isomorphic to \( \mathbb{R}^* \oplus \mathbb{R}^* \).
Exercise 2 (20 points 5-5-5-5):

1) Find $\text{Aut}(\mathbb{Z})$, the group of all automorphisms of the additive group $(\mathbb{Z}, +)$.

2) Let $H$ be a (multiplicative) cyclic group of order 3. Find $\text{Aut}(H)$.

3) Prove that $\text{Aut}(H)$ is isomorphic to $\text{Aut}(\mathbb{Z})$.

4) Let $G$ and $G'$ be two groups such that $\text{Aut}(G)$ is isomorphic to $\text{Aut}(G')$. Are $G$ and $G'$ isomorphic? Justify.
Exercise 3 (20 points 7-8-5): (1) Find $Aut(\mathbb{Q})$, the group of all automorphisms of the additive group $(\mathbb{Q}, +)$.
(2) Prove that $Aut(\mathbb{Q})$ is isomorphic to the multiplicative group $(\mathbb{Q}^*, \times)$.
(3) Prove that the additive group $\mathbb{Q}$ has no proper subgroup of finite index.
Exercise 4 (15 points 5-5-5): Let $G$ be the external direct product of the groups $\mathbb{Z}/3\mathbb{Z}$, $\mathbb{Z}/4\mathbb{Z}$, and $\mathbb{Z}/5\mathbb{Z}$, that is, $G = \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/5\mathbb{Z}$.

(1) Is $G$ a cyclic group?
(2) Is $G$ isomorphic to $\mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/15\mathbb{Z}$?
(3) Is $\mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/5\mathbb{Z}$ isomorphic to $\mathbb{Z}/3\mathbb{Z} \oplus \mathbb{Z}/4\mathbb{Z} \oplus \mathbb{Z}/15\mathbb{Z}$?
Exercise 5 (15 points, 4-7-4):
Let $G = M_n(\mathbb{R})$ be the additive group of all $n \times n$ matrices, $H_1$ the set of all $n \times n$ symmetric matrices (i. e. $A = A^T$) and $H_2$ be the set of all $n \times n$ skew symmetric matrices (i. e. $A = -A^T$).
(1) Prove that $H_1$ and $H_2$ are subgroups of $G$.
(2) Prove that $G$ is the internal direct Product of $H_1$ and $H_2$.
(3) Are $H_1$ and $H_2$ isomorphic?