King Fahd University of Petroleum and Minerals  
Department of Mathematics and Statistics  

MATH 513 Mathematical Methods for Engineers  
Exam 3a Term 122  
Section 01 Instructor: Dr Nadeem Malik  

20th May 2013. Room 5-1001  
Time allowed: 7 – 10 pm (3 hours)  

Important Instructions  

1. Write your name and ID number on each sheet that you use.  
2. At the end of the exam, place all your answer sheets in good order in a bundle. The instructor will staple these together.  
3. Calculators are allowed, but not programmable calculators.  
4. Mobiles must be switched off at all times during the exams; they must be placed in front of the student at all times.  
5. Food is not allowed. Drinks are allowed.  
6. Answer all questions. Part I (short questions) is worth a total of 30 points; Part II (long questions) is worth a total of 80 points.  

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ID:  

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1. Use the separation of variables method to solve Laplace’s equation,

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 4, \quad 0 < y < 8
\]

with

\[
u(x,0) = x, \quad u(x,8) = 4, \quad u(4,y) = 4
\]

\[
u(0,y) = \begin{cases} 
0 & 0 < y < 2 \\
y - 2 & 2 < y < 6 \\
4 & 6 < y < 8 
\end{cases}
\]

[25 points]

2. Solve the wave equation,

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, \quad t > 0
\]

with boundary conditions, \(u(0,t) = u(4,t) = 0\) for \(t > 0\), and initial conditions,

\[
u(x,0) = \begin{cases} 
\frac{x}{4-x} & 0 < x < 2 \\
4 - x & 2 < x < 4
\end{cases}, \quad \text{and} \quad \frac{\partial u(x,0)}{\partial t} = 0 \text{ for } 0 < x < 4.
\]

[25 points]
3. Use the separation of variables method to solve the Laplace equation for $u(r,z)$ in cylindrical coordinates,

$$
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial z^2} = 0, \quad 0 < r < 4, \quad 0 < z < 2
$$

with

$$
u(4, z) = 0
$$
$$
u(r, 0) = 0
$$
$$
u(r, 2) = 8
$$

and the solution remains finite at $r = 0$. [25 points]

4.

(i) If $X(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ for $-\infty < t < \infty$, find the solution to the initial value problem,

$$
X'(t) = \begin{pmatrix} -3/2 & -2 \\ 2 & 5/2 \end{pmatrix} X(t) + \begin{pmatrix} -1/2 \\ 1/2 \end{pmatrix} + \begin{pmatrix} 3/2 \\ -13/2 \end{pmatrix} e^{-2t}
$$

with $X(0) = \begin{pmatrix} 3 \\ -3/2 \end{pmatrix}$. [15 points]

(ii) Let $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

1. Find the matrix $P$ that diagonalises $A$.

2. Find $e^{At}$

3. Find $A^5$ without using explicit matrix multiplications. (No points will be given if you use matrix multiplications.) [10 points]

N Malik