Prob. 1
Let $E$ be a non-empty set and $\mathcal{C} \subseteq \mathcal{P}(E)$ a non-empty collection. Prove that there exists a smallest $\sigma$-algebra on $E$ containing $\mathcal{C}$. 
Prob. 2
Let $f : E \to F$ be an application and $C \subset \mathcal{P}(F)$. Prove that $\sigma_E(f^{-1}(C)) = f^{-1}(\sigma_F(C))$. 
Prob. 3

Let \((E, \Sigma, \mu)\) be a measure space, \(A \in \Sigma\) and \(B \subset E\) such that \(A \Delta B \in \Sigma\) and \(\mu(A \Delta B) = 0\). Prove that \(B \in \Sigma\) and \(\mu(A \cap C) = \mu(B \cap C)\) for all \(C \in \Sigma\).
**Prob. 4** Let $\mu^*$ be an outer measure on $E$ and $A \subset E$. Prove that the following are equivalent

(i) $A$ is $\mu^*$-measurable

(ii) For every $\varepsilon > 0$ there exist $A_1$ and $A_2$, $\mu^*$-measurable, $A_1 \subset A \subset A_2$ such that $\mu^*(A_2 \setminus A_1) < \varepsilon$

(iii) For every $P \subset A$ and $Q \subset A^c$ we have $\mu^*(P \cup Q) = \mu^*(P) + \mu^*(Q)$. 
Prob. 5
Let \((E, \Sigma, \mu)\) be a finite measure space, \(\mu^*\) the outer measure generated by \(\mu\) and \(A \subset E\). Prove that \(A\) is \(\mu^*\)-measurable if and only if \(\mu^*(E) = \mu^*(A) + \mu^*(A^c)\).
Prob. 6

Let $A \in \mathcal{L}$ be such that $0 < m(A) < \infty$. We define the application $f : \mathbb{R}^+ \to \mathbb{R}^+$ by $f(x) := m(A \cap (-x, x))$. Prove that

(i) $f$ is continuous and $f(\mathbb{R}^+)$ is equal to $[0, m(A)]$ or $[0, m(A))$

(ii) For all $c > 0$ there exist $\{A_n\}_{n \geq 1} \subset \mathcal{L}$, $A_n \subset A$, $n = 1, 2...$ such that $\sum_{n \geq 1} m(A_n) = cm(A)$. 


Prob. 7

Let \( \mu^* : \mathcal{P}(E) \to \mathbb{R}^+ \) be an outer measure. (a) Prove that \( |\mu^*(A) - \mu^*(B)| \leq \mu^*(A \Delta B) \) for all \( A, B \subset E \), such that \( \mu^*(A) < \infty \) or \( \mu^*(B) < \infty \).

(b) Let \( A, B \subset E \) be such that \( B \) is \( \mu^* \)-measurable, \( B \subset A \) and \( \mu^*(A) = \mu^*(B) < \infty \). Prove that \( A \) is \( \mu^* \)-measurable.