King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
MATH 536 [Functional Analysis II]
Semester (122)

Exam I: March 16, 2013 Time allowed: 2hrs:

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(Q1) (a)Prove that every orthonormal subset of a Hilbert space is contained in a maximal orthonormal subset of H.

(b) Suppose that Parseval formula holds for each x in a Hilbert space. Use projection theorem to show that any orthonormal set $\{e_i\}$ in H is an orthonormal basis of H.

(Q2) (a) Let H be a Hilbert space and $T \in L(H)$, the space of bounded linear operators from H into H. Define adjoint T^* of T. Hence verify that the left shift operator is adjoint of the right adjoint shift operator on the usual Hilbert space $(l_2, \langle\cdot\rangle)$.

(b) If T is as in part (a), then show that
$\| T \| = \sup\{|\langle Tx, x\rangle| : \| x \| \leq 1\}$ provided T is a self-adjoint.

(Q3) (a) If T is a bounded linear operator on a Hilbert space, then show that T^*T and TT^* are positive operators.
(b) If $T \in L(H)$ [as in Q2(a)] is isometric and onto, then prove that T is unitary.

(Q4) (a) Define strong, weak and weak* topologies on a normed space $(X, \| \cdot \|)$. Describe relationship among these three topologies on X (do not include details). Does weak convergence of $\{x_n\}$ in X imply its weak* convergence in X?

(b) Let X^* be the dual of a normed space X. The closed unit ball $B_1^* = \{f \in X^* : \| f \| \leq 1\}$ is compact under weak* topology. Use this fact to prove that every separable Banach space is isometric to a subspace of $C[0, 1]$.