KING FAHD UNIVERSITY OF PETROLEUM & MINERALS  
Department of Mathematics & Statistics

Math 605  Asymptotic Expansions and Perturbation Methods

Mid Term Exam  Term 122

Time Allowed 2 Hours

Name ________________  ID # ______

<table>
<thead>
<tr>
<th>Q #</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/ 8</td>
</tr>
<tr>
<td>2</td>
<td>/ 10</td>
</tr>
<tr>
<td>3</td>
<td>/ 10</td>
</tr>
<tr>
<td>4</td>
<td>/12</td>
</tr>
<tr>
<td>Total</td>
<td>/ 40</td>
</tr>
</tbody>
</table>

Important Note

Write clearly and show all work.

Instructor: F. D. Zaman
**Q 1)** Define the following notions for \( z \to \infty \)

(i) Order relation \( O \)

(ii) Order relation \( o \)

(iii) \( f \) asymptotic to \( g \)

(iv) Asymptotic sequence
(b) Consider \( f(z) = 2z + z \cos z \). Find \( g(z) \) such that \( f(z) = O(g(z)), \ z \to \infty \)

When \( f(z) \) is purely imaginary and when it is purely real.

(c) Find asymptotic representation of \( f(z) = \frac{1}{z-1}, \ z \to \infty \) in two different ways.
Q2) Consider $E_i(x) = \int_x^\infty \frac{e^{-t}}{t} dt$. Obtain an asymptotic representation of $E_i(x)$ for large $x$. Show that this is indeed an accurate approximation for large $x$. Also show that the series representation is however divergent.
Q3) Use the Laplace method (not the formula) to obtain an asymptotic approximation of the integral

\[ f(x) = \int_{-\infty}^{\infty} \exp(-x \cosh t) dt, \quad x \to \infty. \]
Q4) Find asymptotic approximation for

(a) \[ \int_0^\infty \cos \lambda (t^3 - t) dt, \quad \lambda \to \infty \]
Q 4) (b) \( \int_{0}^{\pi} \cos(x \sin t - nt) dt \), \( x \to \infty \).