King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

CODE 001 — **Math 101** — CODE 001

Exam II

Term 123

Sunday 14/07/2013

Net Time Allowed: 120 minutes

Name: Detailed Key

ID: ______________ Sec: ______________.

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The number of points at which the curve $y = x^4 - 8x^2 + 3$ has horizontal tangents is

$y' = 4x^3 - 16x = 0 \Rightarrow 4x(x^2-4) = 0$

$\boxed{x = 0}$ or $\boxed{x = \pm 2}$ are c.n.

(a) 4

(b) 2

(c) 3

(d) 0

(e) 1

2. If $y = \dfrac{1}{\sec x + \tan x}$ then $\dfrac{dy}{dx}\Big|_{x=0} =$

$$\frac{dy}{dx} = -\frac{\sec x \tan x + \sec^2 x}{(\sec x + \tan x)^2}$$

$$= -\frac{\sec x(\cancel{\tan x + \sec x})}{(\sec x + \tan x)^2}$$

$$= \frac{-\sec x}{\sec x + \tan x}$$

$$\frac{dy}{dx}\Big|_{x=0} = \frac{-1}{1+0} = -1$$

(a) 1

(b) $\frac{1}{2}$

(c) 0

(d) $\frac{-1}{2}$

(e) -1

3. If $f(x) = \sin(\sin^2 x)$, then $f'(x) =$ $\cos(\sin^2 x) \cdot 2 \sin x \cos x$

$= \sin(2x) \cdot \cos(\sin^2 x)$

(a) $2 \sin x \cos x \cdot \sin(\sin^2 x)$

(b) $2 \cos x \cdot \cos(\sin^2 x)$

(c) $2 \sin^2(x) \cdot \cos(\sin^2 x)$

(d) $\sin(2x) \cdot \sin(\cos^2 x)$

(e) $\sin(2x) \cdot \cos(\sin^2 x)$

4. The linearization of $f(x) = \sqrt[3]{8 - 3x}$ at $x = 0$ is given by

(a) $L(x) = 8 + \frac{1}{3}x$

(b) $L(x) = 2 - x$

(c) $L(x) = 2 - \frac{1}{4}x$

(d) $L(x) = 1 + 3x$

(e) $L(x) = 2 - 4x$

$L(x) = f(a) + f'(a)(x-a)$

$L(0) = f(0) + f'(0)x$

$f(x) = (8-3x)^{1/3}$

$f'(x) = -3 \cdot \frac{1}{3}(8-3x)^{-2/3}$

$f'(0) = -(8)^{-2/3} = -\frac{1}{4}$

$f(0) = \sqrt[3]{8} = 2$

$L(x) = 2 - \frac{1}{4}x$

5. The edge of a cube increases at a rate of $3\,cm/s$. When the edge length is $2\,cm$, the rate at which the **surface area** of the cube is increasing is

(a) $40\,cm^2/s$

(b) $72\,cm^2/s$

(c) $12\,cm^2/s$

(d) $36\,cm^2/s$

(e) $84\,cm^2/s$

$\frac{dx}{dt} = 3$, $x = 2$, $\frac{dS}{dt} = ?$

$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$

$\left.\frac{dS}{dt}\right|_{x=2} = 24 \cdot 3 = 72\ cm^2/s$

6. The slope of the tangent line of the curve $y^2 = 1 + x^2 + \sin(xy)$ at the point $(0, 1)$ is

(a) 0

(b) 1

(c) 2

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

$2y\frac{dy}{dx} = 2x + \cos(xy)\left(y + x\frac{dy}{dx}\right)$

$= 2x + y\cos(xy) + x\cos(xy)\frac{dy}{dx}$

$\left[2y - x\cos(xy)\right]\frac{dy}{dx} = 2x + y\cos(xy)$

$\frac{dy}{dx} = \frac{2x + y\cos(xy)}{2y - x\cos(xy)}$

$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{0 + \cos(0)}{2 - 0} = \frac{1}{2} = m$

$y - 1 = m(x - 0) \Rightarrow y - 1 = \frac{1}{2}x$

$\Rightarrow \boxed{y = \frac{1}{2}x + 1}$

7. An equation for the tangent line to the curve $y = (x^2+2)e^x$ when $x = 0$ is given by

(a) $y = -2x + 2$

(b) $y = \frac{1}{2}x + 2$

● $y = 2x + 2$

(d) $y = -x + 3$

(e) $y = x + 2$

$m = y'|_{x=0}$, where

$y' = 2xe^x + (x^2+2)e^x$

$= (x^2+2x+2)e^x$

$y'|_{x=0} = 2e^0 = 2$

$y - y(0) = 2(x-0)$

$y - 2 = 2x \Rightarrow \boxed{y = 2x+2}$

8. If $z = \left(\frac{u-1}{u+1}\right)^2$ and $u = \frac{1}{x^2} - 1$, then $\frac{dz}{dx}|_{x=-1}$ is

(a) 4

● -8

(c) $\frac{1}{2}$

(d) 1

(e) -2

$\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx}$

$= 2\left(\frac{u-1}{u+1}\right) \cdot \frac{\cancel{u}+1-\cancel{u}(+)}{(u+1)^2} \cdot -\frac{2}{x^3}$

$= \frac{4(u-1)}{(u+1)^3}\left(\frac{-2}{x^3}\right)$

$= \frac{4\left(\frac{1}{x^2}-1-1\right)}{\left(\frac{1}{x^2}-1+1\right)^3}\left(\frac{-2}{x^3}\right)$

$= \frac{\frac{4}{x^2} - 8}{\frac{1}{x^6}} \cdot \frac{-2}{x^3}$

$= -2 \frac{\frac{4}{x^2}-8}{\frac{1}{x^3}}$

$= -2(4x - 8x^3)$

$= -8x + 16x^3$

$\frac{dz}{dx}\Big|_{x=-1} = 8 - 16 = -8$

9. If $y = \cos(2\ln x)$, then $x^2y'' + xy' =$

(a) $2y$

(b) $5y$

(c) 0

(d) $-4y$

(e) $\frac{y}{x}$

$y' = -\sin(2\ln x)\cdot\frac{2}{x}$

$y'' = \frac{2}{x^2}\sin(2\ln x) - \frac{4}{x^2}\cos(2\ln x)$

$x^2y'' = \cancel{2\sin(2\ln x)} - 4\cos(2\ln x)$

$+\ xy' = \cancel{-2\sin(2\ln x)} \Rightarrow$

$x^2y'' + xy' = -4\cos(2\ln x) = -4y$

10. At time t the position $s(t)$ of a body moving in a straight line is given by

$$s(t) = t^3 - 6t^2 + 9t \ \text{(in } m\text{)}$$

The **total distance** travelled by the body from $t = 0$ to $t = 4$ is

(a) $8\,m$

(b) $12\,m$

(c) $0\,m$

(d) $4\,m$

(e) $6\,m$

$v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$

$v(t) = 0$ if $t^2 - 4t + 3 = 0 \Rightarrow (t-3)(t-1) = 0$

at $\boxed{t = 1}$ & $\boxed{t = 3}$

distance $= |s(1) - s(0)| + |s(3) - s(1)| + |s(4) - s(3)|$

$s(0) = 0$, $s(1) = 4$

$s(3) = 27 - 54 + 27 = 0$

$s(4) = 64 - 96 + 36 = 4$

distance $= |4 - 0| + |0 - 4| + |4 - 0|$

$= 4 + 4 + 4 = 12$

11. If $\cot\left(\frac{x}{y}\right) = x + y^2$, then using implicit differentiation we get $y' =$

(a) $\dfrac{x\csc^2\left(\frac{x}{y}\right) + y^2}{y\csc^2\left(\frac{x}{y}\right) - x^2}$

(b) $\dfrac{\csc^2\left(\frac{x}{y}\right) + y}{\csc^2\left(\frac{x}{y}\right) - 2y}$

(c) $\dfrac{y - \csc^2\left(\frac{x}{y}\right)}{x + \csc^2\left(\frac{x}{y}\right)}$

(d) $\dfrac{xy - \csc\left(\frac{x}{y}\right)}{y^2 + \csc\left(\frac{x}{y}\right)}$

● $\dfrac{y\csc^2\left(\frac{x}{y}\right) + y^2}{x\csc^2\left(\frac{x}{y}\right) - 2y^3}$

$$-\csc^2\left(\frac{x}{y}\right)\cdot\frac{y - x\frac{dy}{dx}}{y^2} = 1 + 2y\frac{dy}{dx}$$

$$-\frac{1}{y}\csc^2\left(\frac{x}{y}\right) + \frac{x}{y^2}\csc^2\left(\frac{x}{y}\right)\frac{dy}{dx} = 1 + 2y\frac{dy}{dx}$$

$$\frac{x}{y^2}\csc^2\left(\frac{x}{y}\right)\frac{dy}{dx} - 2y\frac{dy}{dx} = 1 + \frac{1}{y}\csc^2\left(\frac{x}{y}\right)$$

$$\frac{dy}{dx}\left(\frac{x}{y^2}\csc^2\left(\frac{x}{y}\right) - 2y\right) = 1 + \frac{1}{y}\csc^2\left(\frac{x}{y}\right)$$

$$\frac{dy}{dx}\,\cancel{\frac{1}{y}}\left(\frac{x}{y}\csc^2\left(\frac{x}{y}\right) - 2y^2\right) = \cancel{\frac{1}{y}}\left(y + \csc^2\left(\frac{x}{y}\right)\right)$$

$$\frac{dy}{dx} = \frac{y + \csc^2\left(\frac{x}{y}\right)}{\frac{x}{y}\csc^2\left(\frac{x}{y}\right) - 2y^2}$$

$$= \frac{y^2 + y\csc^2\left(\frac{x}{y}\right)}{x\csc^2\left(\frac{x}{y}\right) - 2y^3}$$

12. If $y = x^2\sin^{-1}(x^2) + \sqrt{1 - x^4}$, then $y' =$

● $2x\sin^{-1}(x^2)$

(b) $2x\sin^{-1}(x^2) + \dfrac{4x}{\sqrt{1 - x^4}}$

(c) $x\sin^{-1}(x^2) + \dfrac{4x^3}{\sqrt{1 - x^4}}$

(d) $\sin^{-1}(x^2) - \dfrac{2x^3}{\sqrt{1 - x^4}}$

(e) $2x\sin^{-1}(x^2) - \dfrac{2x}{\sqrt{1 - x^4}}$

$$2x\sin^{-1}(x^2) + \frac{2x^3}{\sqrt{1-x^4}} - \frac{\overset{2}{\cancel{4}}x^3}{\cancel{2}\sqrt{1-x^4}}$$

$$= 2x\sin^{-1}(x^2)$$

13. If f is a differentiable function of x and $g(x) = e^x f(e^{-x})$, then $g'(x) =$

(a) $g(x) + f'(e^{-x})$

(b) $e^x f'(e^{-x})$

(c) $e^x(f'(e^{-x}) + f(e^{-x}))$

(d) $f'(e^{-x})$

● $g(x) - f'(e^{-x})$

$$g'(x) = e^x f(e^{-x}) - e^{-x} f'(e^{-x}) e^x$$
$$= e^x f(e^{-x}) - f'(e^{-x})$$
$$= g(x) - f'(e^{-x})$$

14. If $\frac{d}{dx}[f(3x)] = 6x$, then $\frac{d}{dx}[f(x)] =$

● $\frac{2}{3}x$

(b) $2x$

(c) 2

(d) $\frac{3}{2}x$

(e) $3x$

$$\frac{d}{dx}[f(3x)] = 3f'(3x) = 6x$$
$$u = 3x \Rightarrow x = \frac{u}{3}$$
$$3f'(u) = 6 \cdot \frac{u}{3} = 2u$$
$$f'(u) = \frac{2}{3}u$$

15. Using a suitable linear approximation, the value of $\ln(1.02)$ is approximated by

- (a) 0.02
- (b) 0.01
- (c) 1.02
- (d) 1.01
- (e) 0.04

$f(x) \approx f(a) + f'(a)(x-a)$

let $f(x) = \ln x$, $f'(x) = \frac{1}{x}$

$x = 1.02$, $a = 1 \Rightarrow$

$f(1.02) \approx \ln 1 + \frac{1}{1}(1.02 - 1)$

$\approx 0 + 0.02$

$\ln(1.02) \approx 0.02$

16. If $y = \sqrt[3]{\dfrac{x(x+1)(x+2)}{(x+3)(x+4)(x+5)}}$, then $y'|_{x=1} =$

- (a) $\dfrac{73}{60\sqrt[3]{20}}$
- (b) $\dfrac{61}{170\sqrt[3]{20}}$
- (c) $\dfrac{52}{61\sqrt[3]{23}}$
- (d) $\dfrac{43}{3\sqrt[3]{30}}$
- (e) $\dfrac{73}{180\sqrt[3]{20}}$

let $\ln y = \frac{1}{3}\left[\ln x + \ln(x+1) + \ln(x+2) - \ln(x+3) - \ln(x+4) - \ln(x+5)\right]$

$\frac{y'}{y} = \frac{1}{3}\left(\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4} - \frac{1}{x+5}\right)$

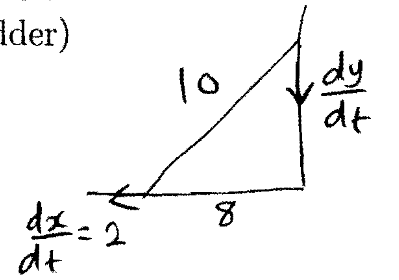
$y' = \frac{1}{3} y \left(\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3} - \frac{1}{x+4} - \frac{1}{x+5}\right)$

$y'|_{x=1} = \frac{1}{3} y(1)\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{5} - \frac{1}{6}\right)$

$y'|_{x=1} = \frac{1}{3}\sqrt[3]{\frac{6}{120}} \cdot \left(\frac{60+30+20-15-12-10}{60}\right)$

$= \frac{1}{3}\frac{1}{\sqrt[3]{20}} \cdot \frac{73}{60} = \frac{73}{180\sqrt[3]{20}}$

17. A $10-ft$ ladder is leaning against a vertical wall when its base starts to slide away from the wall. By the time the base is $8\,ft$ from the wall, the base is moving at the rate of $2\,ft/s$. At this instant, the rate at which **the area of the triangle** (formed by the wall, the ground, and the ladder) is changing is



(a) $-\frac{14}{3}\,ft^2/s$

(b) $-\frac{25}{2}\,ft^2/s$

(c) $\frac{5}{2}\,ft^2/s$

(d) $\frac{25}{2}\,ft^2/s$

(e) $6\,ft^2/s$

$A = \frac{1}{2} x y$

$100 = x^2 + y^2$

$y = \sqrt{100 - x^2}$

$A = \frac{1}{2} x \sqrt{100 - x^2}$

$\frac{dA}{dt} = \frac{1}{2}\sqrt{100-x^2}\,\frac{dx}{dt} - \frac{x}{2}\,\frac{2x\,\frac{dx}{dt}}{2\sqrt{100-x^2}}$

$\left.\frac{dA}{dt}\right|_{x=8} = \frac{1}{2}6(2) - 4\,\frac{(8)(2)}{6}$

$= 6 - \frac{32}{3} = \frac{18-32}{3} = \frac{-14}{3}\,ft^2/s$

18. An equation for the tangent line to the curve $y = \sqrt{x}$ that passes through the point $(-4, 0)$ is

(a) $3x - 4y = -12$

(b) $x + y = 2$

(c) $x + 4y = -4$

(d) $2x - y = -8$

(e) $x - 4y = -4$

$y' = \frac{1}{2\sqrt{x}}$, $P(x_0, y_0)$

$m = \frac{1}{2\sqrt{x_0}}$, $m = \frac{y_0 - 0}{x_0 + 4}$ $\Rightarrow$

$\frac{1}{2\sqrt{x_0}} = \frac{y_0}{x_0+4} \Rightarrow \frac{1}{2\sqrt{x_0}} = \frac{\sqrt{x_0}}{x_0+4}$

$2x_0 = x_0 + 4 \Rightarrow \boxed{x_0 = 4}, \boxed{y_0 = 2}$

$y - 2 = \frac{1}{2\sqrt{4}}(x - 4)$

$y - 2 = \frac{1}{4}x - 1 \Rightarrow$

$\boxed{y = \frac{1}{4}x + 1}$ or $4y = x + 4$

$4y - x = 4$

19. If $f(x) = (\ln x)^{\tan x}$, then

(a) $f'(x) = \tan x (\ln x)^{\tan x} - (\sec^2 x)\ln x$

(b) $f'(x) = \dfrac{\tan x (\ln x)^{\tan x}}{x \ln x} + (\sec^2 x)\ln x$

(c) $f'(x) = \dfrac{\tan x (\ln x)^{\tan x}}{x \ln x} + (\sec^2 x)\ln(\ln x)(\ln x)^{\tan x}$

(d) $f'(x) = \tan x (\ln x)^{\tan x} - (\sec^2 x)\ln(\ln x)$

(e) $f'(x) = \dfrac{\tan x (\ln x)^{\tan x}}{\ln x} + (\sec^2 x)\ln(\ln x)(\ln x)^{\tan x}$

20. If the line $y = 2x + 3$ is perpendicular to the tangent lines to the curve $y = \dfrac{x}{x-2}$ at the points (a, b) and (c, d), then $a + b + c + d =$

(a) 6

(b) 0

(c) 4

(d) 2

(e) 1

$$y' = \frac{x-2-x}{(x-2)^2} = -\frac{2}{(x-2)^2} = -\frac{1}{2}$$

$$\Rightarrow \frac{4}{(x-2)^2} = 1 \Rightarrow (x-2)^2 = 4$$

$$x - 2 = \pm 2 \Rightarrow \boxed{x = 0} \text{ or } \boxed{x = 4}$$

$$(0, 0), (4, 2)$$

$$a + b + c + d = 0 + 0 + 4 + 2 = 6$$

By logarithmic differentiation:

$$\ln f(x) = \tan x \ln(\ln x)$$

$$\frac{f'(x)}{f(x)} = \sec^2 x \ln(\ln x) + \frac{\tan x}{\ln x} \cdot \frac{1}{x}$$

$$f'(x) = (\ln x)^{\tan x}\left[\sec^2 x \ln(\ln x) + \frac{\tan x}{x \ln x}\right]$$

$$=$$