1. (a) Which of the points $A\,(0,0,0)$ and $B\,(1,1,1)$ is closer to the plane $3x+2y+z=4$?

(b) Find the coordinates of the point P on the plane $2x+y-2z=1$ closest to the point $(1,0,-1)$.

2. Find the area of the region outside the circle $r = \dfrac{1}{\sqrt{2}}$ and inside the lemniscate $r^2 = \sin 2\theta$.

3. Let $f(x,y) = \cos(xy)$. Find an upper bound for the magnitude of the error $|E|$ in the approximation $f(x,y) \approx L(x,y)$ over the square $R : \left|x - \frac{3}{2}\right| \leq \frac{1}{2}, \quad \left|y - \frac{3}{2}\right| \leq \frac{1}{2}$, where $L(x,y)$ is the linearization of $f(x,y)$ at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

4. Find the average value of $f(x, y) = xe^{(2-y)^2}$ over the region in the first quadrant bounded above by the curve $y = 2 - x^2$ and below by the line $y = 1$.

5. Convert the integral $\int_0^1 \int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} (x+y+z)\, dydxdz$ to an equivalent integral in cylindrical coordinates and evaluate it.

6. The surface consisting of all points P such that the distance from P to the x-axis is three times the distance from P to the yz-plane is

(a) an elliptic paraboloid

(b) a cone

(c) an ellipsoid

(d) a hyperboloid of one sheet

(e) a hyperbolic paraboloid

7. Which of the following statements is **true** for the function $f(x, y) = x^2 + y^3 - 3y + 1$?

(a) f has one local maximum and one local minimum

(b) f has one local maximum and one saddle point

(c) f has two local maxima

(d) f has one local minimum and one saddle point

(e) f has two saddle points

8. The volume of the region below the surface $z = x^2 - 2xy + 3$ and above the rectangle

$$R = \{(x, y) \mid 0 \leq x \leq 1, \quad -1 \leq y \leq 1\}$$

is equal to

(a) $\frac{20}{3}$

(b) $\frac{16}{3}$

(c) $\frac{10}{3}$

(d) $\frac{8}{3}$

(e) $\frac{2}{3}$

9. Let $f(x, y) = \left(\tan^{-1}\left(\frac{x}{y}\right)\right)^2$. The slope of the line tangent to this surface at the point $\left(\sqrt{3}, -1\right)$ and lying in the plane $y = -1$ is

(a) $\frac{2\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $-\frac{\pi}{6}$

(d) $\frac{\pi}{3}$

(e) $-\frac{\pi}{3}$

10. The value of $\int_0^3 \int_{\sqrt{\frac{y}{3}}}^1 e^{-x^3} dx dy$ is

(a) $\frac{1}{e}$

(b) $1 + \frac{1}{e}$

(c) $\frac{2}{e}$

(d) $1 - \frac{2}{e}$

(e) $1 - \frac{1}{e}$

11. Let $f(x,y) = \begin{cases} \frac{x^8 + 9y^8}{x^4 + 3y^4} & \text{if } (x,y) \neq (0,0) \\ m & \text{if } (x,y) = (0,0). \end{cases}$ Then

(a) f is continuous at $(0,0)$ if $m = -3$

(b) f is continuous at $(0,0)$ if $m = 1$

(c) f is continuous at $(0,0)$ if $m = 0$

(d) f is continuous at $(0,0)$ if $m = \sqrt{3}$

(e) there is no real value of m for which f is continuous at $(0,0)$

12. Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{3}x$, $y = x$, $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The value of $\iint\limits_R \dfrac{dA}{x^2 + y^2}$ is

(a) $\dfrac{\pi \ln 2}{12}$

(b) $\dfrac{\pi \ln 2}{8}$

(c) $\dfrac{\pi \ln 2}{6}$

(d) $\dfrac{\pi \ln 2}{10}$

(e) $\dfrac{\pi \ln 2}{15}$

13. Let D be the region bounded by the parabola $y = x^2$ and the line $y = 4$. The absolute minimum value of $f(x, y) = 1 + 2xy - 24x + 9y$ over D is

(a) -69

(b) -24

(c) 5

(d) -12

(e) 0

14. Let L and m be respectively the maximum value and the minimum value of $f(x,y) = e^{xy}$ on the curve $4x^2 + y^2 = 2$. The value of $\dfrac{L}{m}$ is

(a) 1

(b) $\dfrac{1}{e}$

(c) e

(d) $2\sqrt{e}$

(e) $\dfrac{2}{\sqrt{e}}$

15. The integral $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{(1-\cos\phi)/2} \rho^2 \sin\phi d\rho d\phi d\theta$ is equal to

(a) π

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{4}$

(e) $\dfrac{\pi}{6}$

16. The average value of the function $f(x, y, z) = 2x + y + z$ over the solid region $R : 1 \leq x \leq 2$, $0 \leq y \leq 1$, $-1 \leq z \leq 1$ is equal to

(a) 1

(b) $\frac{3}{2}$

(c) $\frac{5}{2}$

(d) 3

(e) $\frac{7}{2}$

17. Which of the following pairs are polar coordinates for the same point on the plane?

(a) $(1, \pi/3)$, $(-1, 2\pi/3)$

(b) $(2, \pi/6)$, $(2, 7\pi/6)$

(c) $(1, 17\pi/4)$, $(-1, \pi/4)$

(d) $(2, \pi/6)$, $(-1, \pi/6)$

(e) $(1, 9\pi/4)$, $(-1, 5\pi/4)$

18. The area of the triangle determined by the points $P(m,1,0)$, $Q(1,1,1)$, $R(0,-1,3)$ is equal to $\frac{3}{2}$. The values of m are

(a) 1 and $-\frac{1}{2}$

(b) 2 and $\frac{1}{2}$

(c) -2 and 0

(d) -1 and $\frac{3}{2}$

(e) 3 and $-\frac{3}{2}$

19. The area of the region enclosed by the x-axis and the curve $x = t^3$, $y = 2t^2 + 1$; $-1 \leq t \leq 1$ is equal to

(a) $\frac{22}{5}$

(b) $\frac{27}{5}$

(c) $\frac{32}{5}$

(d) $\frac{17}{5}$

(e) $\frac{12}{5}$