

Ex1. Find an equation for the plane that passes through $(1, 2, 3)$ and parallel to both $u = 2i + 3j + k$ and $v = i - j + 2k$

Solution: The vector $u \times v$ is normal to the plane.

$$u \times v = -2k - 4j - 3k + 6i + j + i = 7i - 3j - 5k$$

An equation of the plane is given by:

$$7(x-1) - 3(y-2) - 5(z-3) = 0$$

$$\text{or } 7x - 3y - 5z = -14$$

Ex2. Find parametric equations for the line in which the planes $x + 2y + z = 1$ and $x - y + 2z = -8$ intersect

Solution: Let $P(x, y, z)$ be a random point in the line of intersection of the two plane. Then the coordinates of P must verify:

$$\begin{cases} x + 2y + z = 1 & (1) \\ \text{and} \end{cases}$$

$$\begin{cases} x - y + 2z = -8 & (2) \end{cases}$$

(2) implies: $y = x + 2z + 8$. Substituting the latter in (1) implies:

$$x + 2(x + 2z + 8) + z = 3x + 5z + 16 = 1$$

$$\text{ie } 3x = -15 - 5z \text{ ie } x = -5 - \frac{5}{3}z.$$

Thus:

$$y = x + 2z + 8 = -5 - \frac{5}{3}z + 2z + 8 = 3 - \frac{1}{3}z.$$

Therefore parametric equations of the line can be:

$$x = -5 - \frac{5}{3}t, \quad y = 3 - \frac{1}{3}t, \quad z = t \text{ with } t \in \mathbb{R}.$$