

Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals

AS381: Actuarial Contingencies I
Dr. Mohammad H. Omar
Major 1 Exam Term 131 FORM A
Wed Oct 2 2013
7.15pm-8.45pm

Name _____ ID#: _____ Serial #: _____

Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the **cheating rules** of the University.
2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financail calculators only. Write important steps to arrive at the solution of the following problems.

The test is 90 minutes, GOOD LUCK, and you may begin now!

Question	Total Marks	Marks Obtained	Comments
1	4		
2	2+4=6		
3	10		
4	6+4=10		
5	1+4=5		
6	1+4=5		
Total	40		

Extra blank page

- 1 (4 points) An insurer with net worth 100 has accepted (and collected the premium for) a risk X with the following probability distribution:

$$\Pr(X = 0) = \Pr(X = 64) = \frac{1}{2}$$

What is the maximum amount G he should pay another insurer to accept 100% of this loss? (Hint: Assume the first insurer's utility function of wealth is $u(w) = \log w$)

Solution:

2. (2+4=6 points) Let the loss random variable X have a p.d.f. given by

$$f(x) = 0.02e^{-0.02x} \quad x > 0$$

a. Calculate $E[X]$.

b. Calculate the pure premium for the two policies below:

$$(i) \quad I(x) = \frac{x}{2}$$

and $(ii) \quad I_d(x) = \begin{cases} 0 & x < d \\ x - d & x \geq d, \text{ where } d = 50 \ln 2. \end{cases}$

(Hint: you may use $E[X - d] = \int_d^\infty (1 - F(x))dx$)

Solution:

3. (10 points) Independent random variables X_k for three lives have the discrete probability functions given below

x	$\Pr(X_1 = x)$	$\Pr(X_2 = x)$	$\Pr(X_3 = x)$
0	0.6	0.5	0.6
1	0.4	0.2	0.0
2		0.3	0.4

Use a convolution process on the non-negative integer values of x to obtain $F_s(x)$ for $x = 0, 1, 2, \dots, 5$ where $S = X_1 + X_2 + X_3$.

Solution (Be sure to show example of similar calculations):

x					$F_s(x)$
0					
1					
2					
3					
4					
5					
Sample calculations					
.					
.					
.					
.					
.					

4. (6+4=10 points) A fire insurance company covers 100 structures against fire damage up to an amount stated in the contract. The numbers of contracts at the different contract amounts are given below.

Benefit Amount (in unit of 10,000)	Number of Contracts
2	80
10	15
20	5

For each structure of the 100 structures, the probability of a claim (assumed to be mutually independent) within a year is 0.06 and the probability of more than one claim is 0. Let S be the amount of claims in a 1 year period.

- Calculate the **mean** and **variance** of S .
- Find the **relative security loading**, θ , so the company can collect an amount equal to the 99th percentile of the distribution of total claims? (Hint: Use normal approximation)

Solution:

5. (1+4=5 points) Given $S(x) = \left(1 - \frac{x}{100}\right)^{1/2}$, for $0 \leq x \leq 100$, calculate the probability that a life age 19 will die between ages 36 and 75.
- a) Less than 0.10
 - b) At least 0.10, but less than 0.20
 - c) At least 0.20, but less than 0.30
 - d) At least 0.30, but less than 0.40
 - e) At least 0.40

Work Shown (4 points)

Hence the answer is .(____)

6. (1+4 = 5 marks) You are given :
- i) ${}^{\circ}e_{20} = 27.5$
 - ii) $l_x = \omega - x, 0 \leq x \leq \omega$
 - iii) $T(x)$ is the future lifetime random variable.

Calculate $Var(T(20))$.

- a) 300
- b) 252
- c) 208
- d) 133
- e) 65

Work Shown (4 points)

Solution:

Hence the answer is .(____)

END OF TEST PAPER